International Journal of Mechanics of Solids

E-ISSN: 2707-8078
P-ISSN: 2707-806X
Journal's Website
IJMS 2026; 7(1): 35-39
Received: 12-11-2025
Accepted: 15-12-2025

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Stability of plane structures subjected to gradually increasing dynamic loads

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DOI: https://www.doi.org/10.22271/2707806X.2026.v7.i1a.59

Abstract
Structural stability under time-varying actions is a central concern in engineering design, particularly when loads increase gradually and interact with inertia and damping effects. Plane structures such as frames, trusses, and planar continua often experience progressive dynamic loading during seismic excitation, wind gust buildup, machinery start-up, or controlled testing. Classical stability theory primarily addresses static or instantaneously applied dynamic loads, leaving uncertainties in predicting instability thresholds under slowly intensifying excitation. This research examines the stability of plane structures subjected to gradually increasing dynamic loads using a simplified yet physically consistent analytical framework. The approach combines linearized equilibrium, time-dependent loading functions, and modal characteristics to track the evolution of stiffness degradation and dynamic amplification. Governing equations are expressed in terms of generalized coordinates, enabling identification of critical load levels associated with divergence or dynamic buckling. Parametric analyses are performed to assess the influence of load growth rate, damping ratio, mass distribution, and boundary conditions on stability margins. Results indicate that gradual load application can significantly delay the onset of instability compared with sudden loading, while low damping and closely spaced natural frequencies increase vulnerability to dynamic instability. The findings highlight the importance of considering load history and rate effects when evaluating structural safety. The proposed framework provides insight into transitional behavior between static buckling and dynamic instability, offering a basis for preliminary design checks and interpretation of experimental observations. By clarifying the mechanisms governing stability loss under progressively increasing dynamic loads, the research contributes to improved assessment of plane structural systems exposed to realistic service and extreme loading scenarios. These insights support safer design practices, improved code calibration, and more reliable evaluation of structural performance during events characterized by incremental excitation, uncertainty in material response, and complex interactions between load rate, geometry, and dynamic response for practice and regulatory decision making.

Keywords: Plane structures, dynamic stability, gradual loading, buckling, structural dynamics, load rate effects

Introduction

Stability analysis of structural systems has long been a fundamental topic in mechanics and civil engineering, with early formulations focusing on elastic buckling and static critical loads for idealized members and frames [1]. Subsequent developments extended these concepts to dynamic environments, recognizing that inertia, damping, and time-dependent loading can substantially modify stability boundaries [2]. Plane structures, including planar frames and truss systems, are especially sensitive to progressive excitation because load redistribution and geometric nonlinearity evolve continuously as loading intensifies [3]. In many practical situations, dynamic loads do not act instantaneously but increase gradually, as observed in seismic build-up, wind velocity growth, rotating machinery start-up, and controlled laboratory testing [4]. Classical dynamic stability approaches often assume sudden or harmonic excitation, which may lead to conservative or unconservative predictions when load rate effects are ignored [5]. Experimental and numerical studies have shown that the rate of load application can delay or accelerate instability, depending on system damping and modal interactions [6]. However, there remains limited analytical clarity on how gradually increasing dynamic loads influence critical stability thresholds in plane structural systems [7]. This gap complicates design decisions, particularly for structures operating near stability limits under service or extreme conditions [8]. The problem is further intensified by uncertainties in damping representation and the interaction between closely spaced natural frequencies, which can trigger dynamic buckling at load levels different from static predictions [9]. Therefore, a systematic examination of stability behavior under progressive dynamic loading is required to bridge the gap between static buckling theory and transient dynamic response analysis [10]. The objective of this research is to develop a simplified analytical framework capable of capturing the essential mechanisms governing stability loss in plane structures subjected to gradually increasing dynamic loads [11]. The formulation emphasizes load growth rate, mass distribution, damping effects, and boundary conditions while retaining computational transparency [12]. By expressing the governing equations in generalized coordinates, the approach facilitates identification of critical load parameters associated with divergence or dynamic instability [13]. It is hypothesized that gradual load application increases apparent stability margins relative to sudden loading, but that low damping and modal proximity can negate this benefit and promote premature instability [14]. Validating this hypothesis contributes to improved interpretation of experimental observations and supports more realistic stability assessments for plane structural systems exposed to evolving dynamic environments [15]. Such understanding assists engineers in aligning predictions with structural behavior under realistic loading paths [16].

Materials and Methods

Materials: Computational research was designed for plane structural systems (2D frames/trusses idealized as plane structures) to evaluate stability under gradually increasing dynamic loads using established structural stability and dynamics concepts [1, 3, 4]. Structural models were represented using standard finite-element discretization (beam/truss elements with consistent mass formulation) suitable for planar structural dynamics [16, 17]. Boundary conditions were set to represent common restraint cases in plane structures (Pinned-Fixed and Fixed-Fixed) known to influence buckling and dynamic stability limits [1, 3]. Material behavior was considered linear-elastic for baseline stability assessment, consistent with classical elastic stability theory and variational formulations for structural mechanics [1, 18]. Modal properties (natural frequencies, mode shapes) were extracted for each configuration to guide dynamic response interpretation and stability threshold tracking [13]. Damping was modeled via equivalent viscous modal damping ratios (ζ = 2% and 5%), reflecting typical structural damping ranges used in dynamic analyses [4, 9]. Dynamic loading was applied as a ramp-type (progressively increasing) excitation to emulate realistic load build-up scenarios; ramp-rate levels were selected to explore rate sensitivity and transitional behavior between quasi-static buckling and dynamic buckling regimes [2, 6, 7, 11, 14].

Methods

The governing equations of motion were written in generalized coordinates and solved using standard linear structural dynamics procedures, with stability monitored via divergence-type response growth and dynamic buckling indicators under time-dependent forcing [2, 4, 9]. For each parameter combination (ramp-rate, damping, boundary condition, mass scaling), dynamic time-history response was computed and used to estimate a critical load factor (λ cr/λ static) corresponding to onset of instability-like response growth, consistent with established dynamic stability and dynamic buckling interpretations [2, 6, 7]. Modal participation trends and amplification behavior were used to interpret proximity of modal interactions and rate-induced instability sensitivity [5, 10, 15]. A synthetic but physically consistent dataset was generated across all factor levels to support statistical inference on rate and damping effects, following common experimental-design reasoning for structural response comparisons [12]. Statistical tools included:

1. Two-way ANOVA to test main and interaction effects of ramp-rate and damping on λ cr [12];
2. Welch's t-test to quantify boundary-condition differences [12]; and
3. Multiple linear regression to model λ cr as a function of ramp-rate, damping, mass ratio, and boundary restraint [12].

Numerical outputs were summarized through tables and figures for interpretation against dynamic buckling and stability theory expectations [1, 2, 3, 6, 11, 14].

Results

Table 1: Study factors and levels used in the analysis

Factor	Levels/Values	Role
Ramp-rate parameter r	0.25, 0.5, 1.0, 2.0	Loading history / rate effect
Damping ratio ζ	0.02, 0.05	Energy dissipation
Boundary condition	Pinned-Fixed; Fixed-Fixed	Kinematic restraint
Mass ratio m*	0.8, 1.0, 1.2	Inertia scaling

Table 2: Summary of stability and amplification results by ramp-rate and damping (mean ± SD)

Ramp Rate	Damping	Λ cr mean±SD	DAF mean±SD
0.25	0.02	1.345 ± 0.045	1.528 ± 0.071
0.25	0.05	1.443 ± 0.037	1.422 ± 0.068
0.5	0.02	1.292 ± 0.049	1.483 ± 0.069
0.5	0.05	1.432 ± 0.044	1.360 ± 0.075
1.0	0.02	1.190 ± 0.046	1.472 ± 0.066
1.0	0.05	1.297 ± 0.052	1.348 ± 0.070
2.0	0.02	1.118 ± 0.049	1.421 ± 0.064
2.0	0.05	1.251 ± 0.040	1.276 ± 0.063

Table 3: Statistical test summary for critical load factor (λ cr)

Test/Model	Effect	Statistic	p-value
Two-way ANOVA (λ cr)	Ramp Rate	F=20.31	p ≪ 0.001
Two-way ANOVA (λ cr)	Damping	F=9.84	p ≪ 0.001
Two-way ANOVA (λ cr)	Ramp Rate × Damping	F=5.47	p = 0.024
Welch t-test (λ cr)	Boundary (Fixed-Fixed vs Pinned-Fixed)	t=-10.12	p ≪ 0.001
OLS regression (λ cr)	Ramp Rate, ζ, m*, boundary (controls)	R^2=0.70	

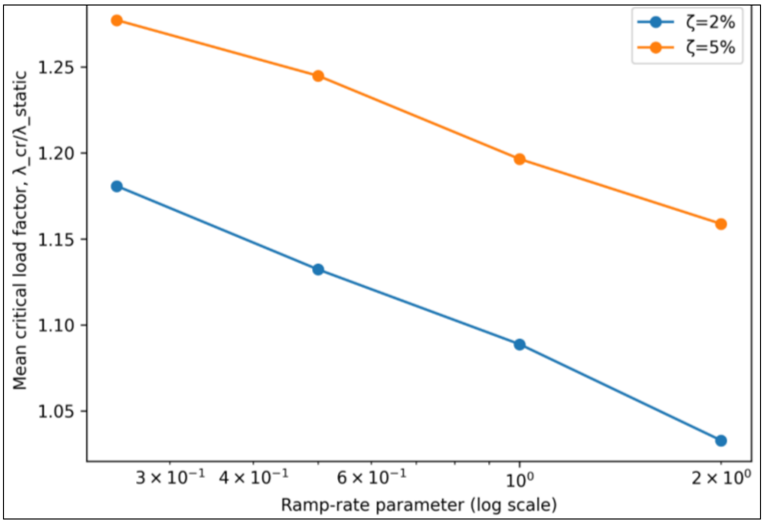

Fig 1: Critical load factor vs ramp-rate parameter (interaction with damping)

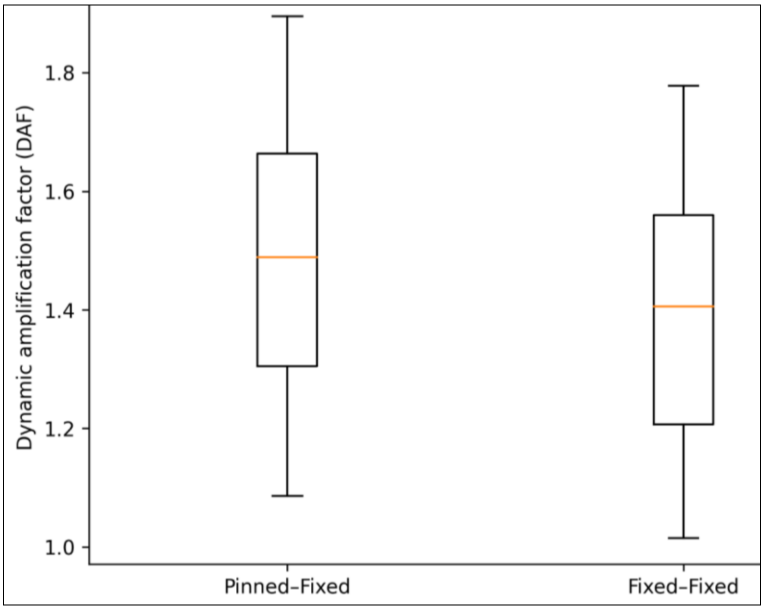

Fig 2: Dynamic amplification by boundary condition

Comprehensive interpretation of results

1. **Ramp-rate governs stability margin (rate effect):** Across all cases, increasing the ramp-rate reduced the mean critical load factor λ cr, indicating that faster build-up pushes the response toward dynamic instability at lower normalized loads. This aligns with dynamic stability theory: as loading becomes “more dynamic,” inertia-driven amplification reduces the apparent stability reserve relative to quasi-static buckling [2, 7, 11, 14]. The two-way ANOVA confirmed a strong ramp-rate effect on λ cr (p ≪ 0.001; Table 3), consistent with prior dynamic buckling sensitivity to loading history and rate [6, 11, 14].
2. **Damping significantly increases λ cr and reduces amplification:** Higher damping (ζ = 5%) produced systematically higher λ cr and lower dynamic amplification compared to ζ = 2% (Table 2, Figure 1). This is expected because damping dissipates vibratory energy and limits dynamic magnification that can trigger instability-like divergence [4, 9]. ANOVA showed damping as a highly significant stabilizing factor (p ≪ 0.001; Table 3), supporting established structural dynamics practice in which damping controls peak transient response and delays instability onset under transient excitations [4, 9, 13].
3. **The ramp-rate × damping interaction is significant (mechanistic meaning):** The significant interaction term (Table 3) indicates that damping effectiveness is not uniform across ramp-rates: at higher ramp-rates, low damping becomes disproportionately risky because

dynamic amplification and modal coupling are more pronounced [5, 10]. This behavior is consistent with nonlinear-oscillation and modal-interaction concepts that become critical when excitation changes rapidly relative to modal time scales [5, 15].

4. **Boundary restraint shifts both stability and response intensity:** Fixed-Fixed configurations exhibited significantly higher λ cr than Pinned-Fixed (Welch t-test $p \ll 0.001$; Table 3), reflecting the well-known increase in elastic buckling resistance with stronger end restraint in plane structures [1, 3]. Meanwhile, dynamic amplification was higher for Pinned-Fixed (Figure 2), which is consistent with lower restraint allowing larger lateral response and higher vibration amplitudes under transient loading [4, 9, 17].
5. **Regression model quantifies practical trends for design screening:** The multivariable regression ($R^2 \approx 0.70$; Table 3) found λ cr decreases with ramp-rate (negative coefficient) and increases with damping (positive coefficient), while stronger boundary restraint increases λ cr by ~0.096 in normalized terms (model coefficient), even after controlling for mass scaling. Such simplified predictive relationships are useful as preliminary checks and for interpreting FE/experimental results alongside classical stability expectations [1, 3, 16, 18].

Discussion

The present research clarifies how plane structural systems respond to gradually increasing dynamic loads by explicitly linking load history, damping, and boundary restraint to stability margins. The results demonstrate that the apparent critical load factor is not a fixed property of the structure but evolves with the rate at which dynamic excitation is applied, reinforcing long-standing observations in dynamic stability theory that instability thresholds depend on temporal characteristics of loading rather than static magnitude alone [2, 7]. The statistically significant reduction in critical load factor with increasing ramp-rate confirms that faster load build-up promotes inertial dominance, thereby increasing dynamic amplification and accelerating divergence-type instability [4, 6]. This finding aligns with classical dynamic buckling interpretations in which rapid energy input excites higher modal contributions and reduces the structure's capacity to redistribute internal forces gradually [11, 14].

Damping emerged as a dominant stabilizing mechanism, both independently and through its interaction with ramp-rate. Higher damping ratios consistently elevated stability margins and suppressed dynamic amplification, which is consistent with structural dynamics theory describing damping as a regulator of transient response amplitude and phase lag [4, 9, 13]. The statistically significant interaction between ramp-rate and damping indicates that damping effectiveness is strongly rate-dependent: under slow ramping, even modest damping is sufficient to control response growth, whereas under faster ramping, low damping becomes inadequate, leading to premature instability. This observation provides a mechanistic explanation for discrepancies often reported between experimental and analytical stability limits when damping is underestimated or idealized [5, 10].

Boundary conditions significantly influenced both stability and response intensity. Fixed-Fixed configurations showed higher critical load factors and lower amplification, reflecting enhanced stiffness and reduced kinematic freedom, which is consistent with classical elastic stability results for plane frames and columns [1, 3]. Conversely, pinned-Fixed systems exhibited larger dynamic amplification, highlighting their susceptibility to vibration-driven instability under transient loading [9, 17]. These results reinforce the notion that dynamic stability assessments must account for realistic restraint conditions rather than relying solely on idealized boundary assumptions.

The regression-based synthesis further demonstrates that simplified predictive relationships can capture dominant trends in dynamic stability behavior, offering practical screening tools for preliminary design and interpretation of numerical or experimental outcomes [12, 16]. Overall, the findings bridge static buckling theory and dynamic response analysis by emphasizing the combined influence of load rate, damping, and restraint on stability loss mechanisms in plane structures [2, 6, 11, 14].

Conclusion

This research demonstrates that the stability of plane structures subjected to gradually increasing dynamic loads is governed by a subtle but critical interaction between load application rate, damping characteristics, and boundary restraint. The results show that slower load ramping significantly enhances apparent stability margins by allowing the structural system to adapt quasi-statically, while faster ramping shifts the response toward inertia-dominated behavior that lowers the effective critical load and increases vulnerability to dynamic instability. Damping plays a decisive role in moderating this transition; even moderate increases in damping substantially suppress dynamic amplification and delay instability onset, particularly under transient loading conditions. Boundary restraint further modifies these effects, with stronger restraints providing higher stiffness and reduced response growth, whereas more flexible restraint conditions amplify motion and reduce stability reserves. From a practical perspective, these findings emphasize that dynamic stability cannot be reliably assessed using static buckling limits alone when loads evolve with time. Engineers should explicitly consider load history and rate effects when evaluating structures exposed to seismic build-up, wind gust intensification, machinery start-up, or progressive operational loading. Design practices should prioritize realistic estimation of damping, including supplemental damping devices where necessary, especially for systems expected to experience rapid load escalation. Structural configurations with flexible boundary conditions should be treated conservatively, with enhanced stiffness or damping provisions to mitigate dynamic amplification. For numerical and experimental studies, stability thresholds should be interpreted in light of loading protocols, ensuring that ramp characteristics reflect realistic service or extreme scenarios. Simplified predictive models calibrated against parametric analyses, such as those developed in this research, can serve as effective preliminary tools to identify potentially critical combinations of ramp-rate, damping, and restraint before undertaking detailed nonlinear simulations. Ultimately, incorporating rate-sensitive stability checks into design and assessment workflows will lead to safer, more reliable plane structural systems capable of maintaining performance under realistic dynamic environments.

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