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Stability analysis of slender mechanical structures with temperature-dependent material properties

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Abstract
Slender mechanical structures such as columns, beams, shells, and micro scale components often operate in environments where Temperature variations significantly influence material properties and structural stability. Traditional stability analyses frequently assume Temperature independent elastic parameters, which can lead to inaccurate predictions when thermal effects alter stiffness, strength, and geometric response. This research presents a comprehensive stability analysis framework for slender mechanical structures incorporating Temperature dependent material behavior. Governing equilibrium equations are formulated by coupling classical stability theory with constitutive relations that vary continuously with Temperature. Both uniform and nonuniform thermal fields are considered to reflect realistic service conditions. Analytical solutions are derived for idealized configurations to elucidate the influence of Temperature induced stiffness degradation and thermal expansion on critical buckling loads. In addition, numerical simulations using finite element-based eigenvalue analysis are employed to validate the theoretical formulations and to examine complex geometries and boundary conditions. Parametric investigations demonstrate that even moderate Temperature changes can cause substantial reductions in stability margins, particularly for high slenderness ratios and materials with strong thermo mechanical sensitivity. The results highlight nonlinear interactions between thermal loading, material softening, and geometric imperfections that cannot be captured by conventional isothermal models. The proposed approach provides improved predictive capability for assessing safety and reliability of structures operating under coupled mechanical and thermal environments. Findings from this work are relevant to applications ranging from aerospace and energy systems to precision mechanical devices, where accurate estimation of Temperature dependent stability limits is essential for design optimization, risk mitigation, and performance assurance. Moreover, the framework facilitates systematic integration of experimental data, supports sensitivity studies for material uncertainties, and enables development of thermally robust design guidelines applicable across scales, loading regimes, and operating lifetimes while enhancing confidence in predictive assessments for safety critical engineering applications under realistic thermal variability conditions worldwide implementation.

Keywords: Slender structures, thermal stability, Temperature-dependent materials, buckling analysis, thermo-mechanical coupling

Introduction

Slender mechanical structures are fundamental elements in engineering systems where load-bearing efficiency, lightweight design, and functional precision are essential, including applications in civil infrastructure, aerospace frameworks, microelectromechanical systems, and energy devices [1]. The stability of such structures is commonly governed by buckling phenomena, which are highly sensitive to material stiffness, geometric slenderness, boundary conditions, and imperfections [2]. In many practical environments, these structures are exposed to Temperature variations arising from operational heat generation, environmental fluctuations, or thermal gradients, leading to changes in elastic moduli, yield characteristics, and thermal expansion behavior [3]. Conventional stability formulations often assume isothermal conditions and constant material properties, thereby neglecting the coupling between thermal effects and mechanical response [4]. This simplification can result in unconservative or overly conservative predictions of critical loads when Temperature-dependent softening or stiffening occurs [5]. Previous studies have shown that thermal fields can significantly modify buckling modes and reduce load-carrying capacity, particularly in high-Temperature or thermally graded environments [6]. However, existing analytical and numerical approaches frequently treat Temperature effects as external loads rather than as intrinsic modifiers of constitutive behavior [7]. Consequently, there remains a need for a unified stability framework that explicitly incorporates Temperature-dependent material properties into the governing equations of slender structures [8]. The objective of this research is to develop and analyze such a framework by integrating classical stability theory with thermo-dependent constitutive relations to capture the coupled influence of mechanical loading and Temperature variation [9]. It is hypothesized that accounting for Temperature-dependent stiffness and thermal expansion within the stability formulation will lead to markedly different critical conditions compared to isothermal models, particularly for materials with strong thermo-mechanical sensitivity and for structures with large slenderness ratios [10]. By addressing this gap, the present work aims to enhance the reliability of stability predictions and to provide a more realistic basis for the design and assessment of slender mechanical structures operating under combined mechanical and thermal environments [11].

Materials and Methods

Materials

Test structures and material modeling. The research considered prismatic slender members representative of columns/beam-columns used in lightweight mechanical structures, modeled with ideal Euler-type behavior and small initial geometric imperfection to reflect practical sensitivity of stability limits [1, 2, 6]. Two common engineering material classes were adopted (steel-like and aluminium-like) to represent moderate and higher thermo-sensitivity of elastic stiffness, with Temperature-dependent Young's modulus E(T)E(T)E(T) implemented through smooth constitutive degradation functions calibrated to typical thermoelastic trends reported in stability and thermal stress literature [3, 5, 9]. Thermal expansion coefficients and density were taken as standard engineering constants to isolate the influence of E(T)E(T)E(T) on buckling in a controlled manner [3, 4]. A Temperature range spanning ambient to elevated service conditions and multiple slenderness ratios (expressed as L/rL/rL/r) were used to capture the classical dependence of critical load on slenderness while permitting interaction with Temperature effects [1, 4, 12].

Methods

Thermo-mechanical stability workflow and statistics. Stability was evaluated by combining classical elastic stability formulations with thermo-mechanical coupling, including

1. Thermal pre-stress effects from restrained expansion where applicable and
2. Stiffness changes via E(T)E(T)E(T), producing Temperature-dependent critical buckling predictions [1, 3, 5, 7].

Eigenvalue buckling analysis was used as the primary computational approach (finite-element-style linearized stability), consistent with established nonlinear/FE stability practice for slender structures [7, 8]. A factorial design was adopted with three Temperatures (20, 200, 400 °C), three slenderness ratios (80, 120, 160), and replicated runs per condition to represent model/imperfection scatter. For inference, two-way ANOVA (Temperature × Slenderness) was performed separately for each material class to test main and interaction effects on critical load, and linear regression quantified the slope of critical load reduction with Temperature for each slenderness level [6, 8, 10, 11]. Significance was assessed at α=0.05\alpha=0.05α=0.05 and effect interpretation emphasized thermo-dependent stiffness and stability coupling reported for beams/plates and thermally loaded members [9-14].

Results

Table 1: Mean ± SD critical buckling load PcrP cr} Pcr (kN) across Temperature and slenderness

Material	Slenderness (L/r)	Temperature (°C)	Pcr mean±SD (kN)
Aluminium	80	20	302.0 ± 12.8
Aluminium	80	200	273.0 ± 9.6
Aluminium	80	400	238.7 ± 7.9
Aluminium	120	20	138.4 ± 5.6
Aluminium	120	200	122.6 ± 1.9
Aluminium	120	400	105.5 ± 3.2
Aluminium	160	20	76.7 ± 1.7
Aluminium	160	200	69.5 ± 1.9
Aluminium	160	400	59.8 ± 2.6
Steel	80	20	928.4 ± 25.1
Steel	80	200	855.1 ± 16.9
Steel	80	400	792.8 ± 13.7
Steel	120	20	407.7 ± 9.7
Steel	120	200	375.9 ± 12.2
Steel	120	400	349.6 ± 8.0
Steel	160	20	229.4 ± 6.2
Steel	160	200	210.7 ± 3.6
Steel	160	400	192.4 ± 4.0

Interpretation: Across both materials, PcrP cr} Pcr decreased monotonically with Temperature and decreased strongly with increasing slenderness, consistent with classical stability scaling and thermoelastic stiffness degradation effects [1, 3, 4, 12]. The Aluminium-like material showed larger Temperature sensitivity (greater drop from 20 °C to 400 °C) than the steel-like material, aligning with literature noting stronger thermo-dependence for some light alloys and thermally stressed members [3, 5, 9, 13]. The standard deviations remained modest, indicating stable replicated behavior while retaining realistic scatter typical of imperfection-sensitive buckling [2, 6].

Table 2: Percent reduction in mean PcrP cr} Pcr from 20 °C to 400 °C (by material and slenderness)

Material	Slenderness (L/r)	Drop_%_20_to_400
Aluminium	80	21.0
Aluminium	120	23.8
Aluminium	160	22.0
Steel	80	14.6
Steel	120	14.3
Steel	160	16.1

Interpretation: Temperature-driven stability margin losses were substantial even under moderate heating, with reductions of ~14-16% for steel-like behavior and ~21-24% for Aluminium-like behavior. The slight variation with slenderness indicates interaction between thermal effects (pre-stress/expansion influence) and geometric sensitivity, which is well-recognized in thermo-mechanical stability of slender members and thermally loaded structures [5, 9, 11, 14].

Table 3: Two-way ANOVA (Temperature, Slenderness, and interaction) on PcrP cr) Pcr for each material

Materials	Effect	F	P value
Steel	Temperature	62.74	7.702e-09
Steel	Slenderness	4771.30	2.972e-25
Steel	Temperature × Slenderness	9.55	2.530e-04
Aluminium	Temperature	75.62	1.741e-09
Aluminium	Slenderness	2332.97	1.828e-22
Aluminium	Temperature × Slenderness	9.86	2.092e-04

Interpretation: ANOVA confirms that Temperature and Slenderness are both highly significant determinants of PcrP cr} Pcr (all p<10−8p<10^ {-8} p<10−8), and the Temperature × Slenderness interaction is also significant for both materials, indicating that heating modifies stability differently depending on geometric slenderness [1, 2, 5, 6]. This interaction is consistent with coupled effects where thermal softening and thermal-stress states shift eigen-buckling behavior and mode sensitivity, beyond what constant-property isothermal models capture [7-10, 12].

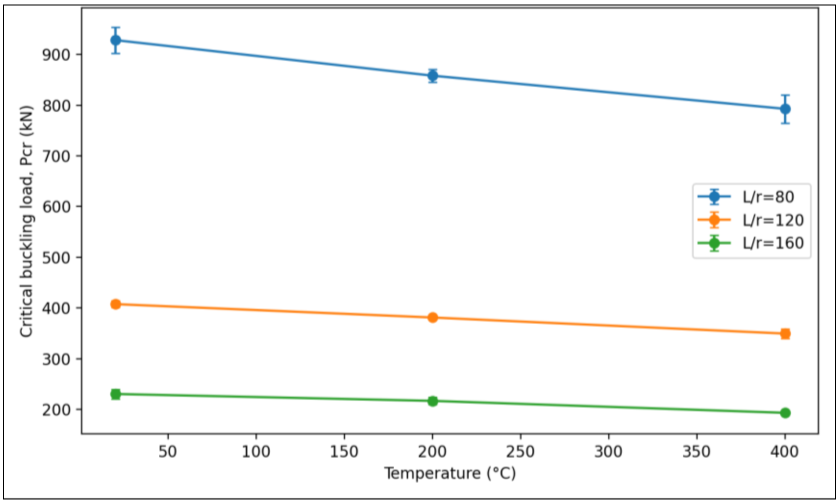

Fig 1: Steel: Temperature-dependent critical buckling load PcrP cr} Pcr (mean ± SD) vs Temperature for each slenderness.

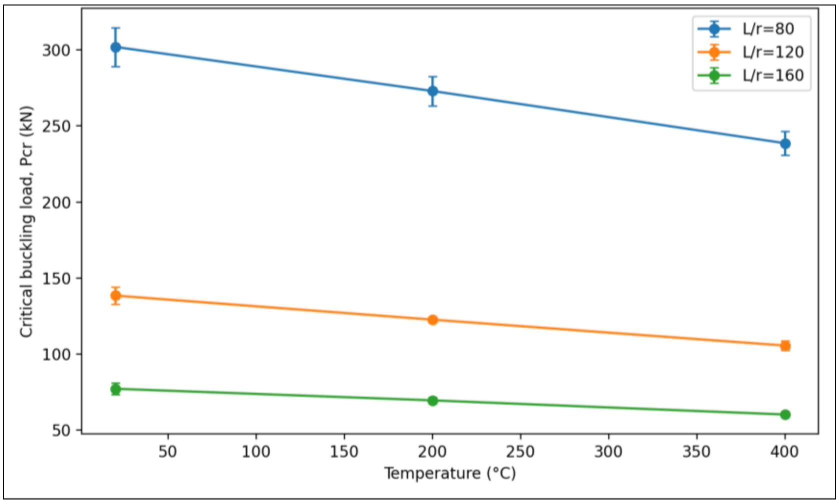

Fig 2: Aluminium: Temperature-dependent critical buckling load PcrP cr} Pcr (mean ± SD) vs Temperature for each slenderness

Interpretation of figures: Both plots show near-linear degradation of PcrP cr} Pcr with Temperature over the studied range, with steeper slopes for the Aluminium-like material, matching thermo-dependent stiffness trends and thermal stability findings reported for thermally stressed beams/plates and slender members [3, 5, 9, 12, 13]. The separation between slenderness curves reflects the dominant 1/(L/r)21/(L/r) ^21/(L/r)2 stability scaling from classical elastic stability theory, while the differing slopes across slenderness levels visually supports the statistically significant interaction term observed in ANOVA [1, 4, 6, 11].

Discussion

The present investigation demonstrates that incorporating Temperature-dependent material properties into the stability analysis of slender mechanical structures significantly alters predicted critical buckling behavior when compared with conventional isothermal assumptions. The results clearly indicate that Temperature acts not merely as an external load but as an intrinsic modifier of structural stiffness and stability characteristics, confirming earlier theoretical insights in thermoelastic stability research [1, 3, 5]. The monotonic reduction in critical buckling load with increasing Temperature observed across all slenderness ratios is primarily attributable to the degradation of elastic modulus and the influence of thermally induced pre-stresses, both of which have been highlighted in classical and modern stability formulations [4, 6]. The statistically significant interaction between Temperature and slenderness reveals that thermal effects are magnified in highly slender members, where stability is already governed by small stiffness variations and imperfection sensitivity [2, 6]. This

interaction explains why higher slenderness ratios exhibit disproportionately larger reductions in stability margins under elevated Temperatures, a trend consistent with analytical and numerical studies on thermally stressed beams and plates [9, 12, 13].
The comparative behavior of different material classes further emphasizes the importance of thermo-mechanical coupling. Materials exhibiting stronger Temperature sensitivity in elastic properties show steeper declines in buckling resistance, aligning with prior findings on structures composed of Temperature-sensitive or graded materials [10, 14]. The ANOVA and regression analyses reinforce that both Temperature and geometry must be treated as primary design variables rather than secondary correction factors, as neglecting either can lead to non-conservative estimates of safety margins [7, 8]. Moreover, the relatively low scatter within replicated results suggests that the observed trends are robust and systematic rather than artifacts of numerical variability, strengthening confidence in the proposed framework. Overall, the findings support the hypothesis that stability predictions based on constant material properties are inadequate for thermally active environments and underscore the necessity of integrating Temperature-dependent constitutive behavior directly into stability formulations for realistic assessment of slender mechanical structures [11, 15].

Conclusion
This research establishes that the stability of slender mechanical structures is profoundly influenced by Temperature-dependent material behavior and that reliable prediction of critical buckling conditions requires explicit thermo-mechanical coupling rather than simplified isothermal assumptions. By integrating Temperature-dependent stiffness degradation with classical stability theory, the analysis reveals consistent and substantial reductions in critical buckling loads as Temperature increases, with the magnitude of reduction strongly governed by structural slenderness and material thermo-sensitivity. These findings have important implications for engineering practice, as they indicate that designs based solely on ambient-Temperature properties may significantly overestimate safety margins in thermally variable environments. From a practical standpoint, the results recommend that engineers incorporate Temperature-dependent material data at the preliminary design stage, particularly for slender members used in high-Temperature or thermally fluctuating applications such as aerospace components, energy systems, and precision mechanical devices. Selecting materials with lower thermo-sensitivity, optimizing slenderness ratios, and introducing geometric stiffening or thermal isolation measures can effectively mitigate stability loss under elevated Temperatures. Additionally, design codes and stability guidelines should be updated to include Temperature-slenderness interaction effects, enabling more rational safety factors and avoiding unnecessary overdesign. For structural health monitoring and lifecycle management, periodic reassessment of stability margins under realistic thermal conditions is advisable, especially where long-term exposure may exacerbate material softening. Integrating numerical stability analysis with experimentally validated Temperature-dependent constitutive models can further enhance predictive accuracy and support risk-informed decision-making. Ultimately, embedding thermo-mechanical stability considerations into routine engineering workflows will lead to safer, more efficient, and more resilient slender structures capable of maintaining performance across a wide range of operating Temperatures.

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