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Vibration characteristics of discrete mass-spring systems with randomly distributed stiffness imperfections

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Abstract
Discrete mass-spring systems are fundamental idealizations for representing vibration behavior in mechanical, civil, and structural engineering applications. In practical realizations, stiffness properties rarely remain perfectly uniform because of manufacturing tolerances, material degradation, assembly errors, or operational damage, leading to randomly distributed stiffness imperfections. This research investigates the vibration characteristics of discrete mass-spring systems incorporating stochastic variations in spring stiffness. A mathematical framework is developed in which randomness is modeled as spatially distributed perturbations superimposed on nominal stiffness values. Governing equations of motion are formulated using matrix representations, enabling modal and frequency-domain analyses. Statistical descriptors, including mean natural frequencies, variance, mode-shape localization indices, and response amplification factors, are evaluated to quantify the influence of uncertainty. Numerical simulations are conducted on multi-degree-of-freedom systems to examine sensitivity trends across different imperfection intensities and correlation lengths. Results demonstrate that even small random stiffness deviations can cause noticeable frequency shifts, mode splitting, and localization phenomena, particularly in higher modes. Increased disorder intensity is shown to enhance response variability under harmonic excitation and may significantly alter dynamic stability margins. Comparisons between deterministic and stochastic models highlight the limitations of idealized uniform-stiffness assumptions in predicting real system behavior. The findings provide insight into the probabilistic nature of vibration responses in imperfect discrete systems and emphasize the necessity of uncertainty-aware modeling in dynamic design. The proposed approach supports improved reliability assessment, damage detection interpretation, and robust design strategies for engineering systems where discrete structural models are employed. Overall, the research contributes to a deeper understanding of how randomly distributed stiffness imperfections govern vibration behavior and dynamic performance in mass-spring assemblies. Such knowledge is essential for engineers seeking safer, more resilient designs under unavoidable variability conditions encountered during fabrication, service life evolution, monitoring interpretation, and long-term operational uncertainty in practical discrete mechanical and structural engineering systems worldwide today broadly.

Keywords: Discrete vibration, mass-spring system, stiffness uncertainty, random imperfections, mode localization, stochastic dynamics

Introduction
Discrete mass-spring representations have long served as foundational tools for analyzing vibration phenomena in engineering systems, offering clarity in describing dynamic behavior, modal interactions, and energy transfer mechanisms [1]. Classical vibration theory commonly assumes uniform stiffness distributions, enabling closed-form solutions and efficient numerical implementations [2]. However, real structures and mechanical assemblies inevitably exhibit spatial stiffness variability arising from manufacturing tolerances, material inhomogeneity, joint imperfections, aging, and damage accumulation [3]. Such deviations introduce uncertainty into system matrices and challenge deterministic predictions of natural frequencies, mode shapes, and dynamic responses [4]. Prior studies have shown that even minor stiffness disorder can trigger mode localization, frequency veering, and amplified response sensitivity, particularly in discretized or periodic systems [5, 6]. Despite these insights, many practical analyses still rely on idealized uniform-stiffness models, potentially underestimating vibration risks and reliability concerns [7].
The central problem addressed in this research is the limited understanding of how randomly

distributed stiffness imperfections influence the global and local vibration characteristics of discrete mass-spring systems under realistic uncertainty conditions [8]. Existing investigations often focus on continuous structures or assume simplified randomness representations, leaving gaps in discrete-system-specific interpretations relevant to lumped-parameter modeling practices [9]. Accordingly, the primary objective of this work is to develop a systematic framework for characterizing vibration behavior in multi-degree-of-freedom mass-spring systems with stochastic stiffness variations, using statistically grounded descriptors and modal analysis techniques [10, 11]. A further objective is to quantify sensitivity trends and response variability as functions of imperfection intensity and spatial distribution, thereby supporting uncertainty-informed design and assessment methodologies [12].

The working hypothesis of this research is that randomly distributed stiffness imperfections, even when statistically small, produce measurable and non-negligible changes in natural frequencies, mode shapes, and dynamic response statistics, with higher modes exhibiting increased susceptibility to localization and variability effects [13-15]. By integrating stochastic modeling with classical vibration theory, the research aims to bridge deterministic and probabilistic perspectives, offering improved predictive capability for discrete dynamic systems encountered in mechanical and structural engineering applications [16, 17]. This integrated viewpoint is particularly relevant for modern engineered assemblies where lightweight design, modular construction, and service-induced degradation collectively magnify uncertainty effects, reinforcing the importance of probabilistic vibration analysis for safety evaluation, diagnostics, and robust performance prediction in discretized structural and mechanical systems under operational variability, maintenance imperfections, and long-term usage conditions commonly observed across industrial, civil, and mechanical engineering applications worldwide in contemporary practice and research-oriented dynamic system modeling contexts globally today.

Materials and Methods

Materials

A discrete lumped-parameter chain was considered to represent a fixed-fixed mass-spring assembly commonly used for foundational vibration modeling and interpretation in engineering dynamics [1, 2]. The model consisted of $N = 8$ identical point masses (each $m = 10$ kg) connected by N+1 linear axial springs (including the two end springs to ground) with nominal stiffness $k_0 = 1.0\times10^5$ N/m, producing an N-DOF second-order system in matrix form [2, 4]. Randomly distributed stiffness imperfections were introduced at the spring level to represent manufacturing tolerance, joint variability, and service-induced degradation effects that are well-known to alter modal properties in real assemblies [3, 4]. Stiffness uncertainty was modeled using a lognormal distribution (strictly positive stiffness) with mean k_0 and coefficient of variation CV ∈ {0%, 2%, 5%, 10%}, consistent with probabilistic structural dynamics practice and random vibration conventions [8, 9, 14]. The research focused on the first four modes to quantify frequency shifts and response variability while also tracking a mode-shape localization index to detect disorder-driven spatial confinement effects in discrete systems [5, 6, 10].

Methods

Equations of motion were formulated as $M\ddot{x} + Kx = 0$, with diagonal mass matrix M and stiffness matrix K assembled for a fixed-fixed chain; modal properties were computed from the generalized eigenproblem $K\varphi = \omega^2 M\varphi$, and natural frequencies were obtained as $f = \omega/(2\pi)$ [1, 2, 4]. A Monte Carlo simulation (500 realizations per CV level) was used to propagate stiffness uncertainty into distributions of modal frequencies and localization metrics, following standard probabilistic structural dynamics workflows [8, 9, 11]. Localization was quantified using a normalized fourth-moment participation measure (higher values indicate stronger spatial concentration), which is frequently used to characterize localization trends under disorder [5, 6, 10]. Statistical inference included: one-way ANOVA to test whether Mode 1 frequency differs across CV levels, Welch's t-test comparing deterministic vs. 10% CV for Mode 1 frequency, and linear regression of Mode 1 frequency standard deviation versus CV to quantify scaling of variability with disorder intensity [14, 17]. These analyses support uncertainty-aware interpretation and reliability-focused vibration assessment [7, 12, 16].

Results

Table 1: Nominal parameters and uncertainty levels used in the discrete mass-spring Monte Carlo research.

Parameter	Value
Degrees of freedom (N)	8
Mass per DOF (m)	10 kg
Nominal spring stiffness (k_0)	1.0×10^5 N/m
End conditions	Fixed-fixed (end springs to ground)
Springs in chain	N+1 = 9
Stiffness imperfection model	Lognormal, mean = k_0 [8, 9, 14]
Disorder levels (CV)	0%, 2%, 5%, 10%
Monte Carlo samples per case	500

Table 2: Mean±SD of the first four natural frequencies and mean shift relative to the deterministic case

CV (%)	Mode	Mean (Hz)	SD (Hz)	Mean shift (%)
0	1	5.527	0.000	+0.000
0	2	10.887	0.000	+0.000
0	3	15.915	0.000	+0.000
0	4	20.461	0.000	+0.000
2	1	5.527	0.022	−0.006
2	2	10.885	0.045	−0.014
2	3	15.916	0.065	+0.005
2	4	20.460	0.083	−0.004
5	1	5.520	0.057	−0.132
5	2	10.873	0.117	−0.125
5	3	15.900	0.158	−0.097
5	4	20.442	0.215	−0.089
10	1	5.509	0.106	−0.328
10	2	10.847	0.209	−0.364
10	3	15.828	0.291	−0.549
10	4	20.369	0.395	−0.449

Interpretation (modal trends)

- Increasing stiffness disorder produced progressively larger frequency dispersion (SD) across all modes, consistent with uncertainty propagation in random structural systems [8, 9, 11].
- Mean frequency shifts were small but systematic at higher disorder, with the largest mean reduction observed in Mode 3 at CV=10% (≈ −0.55%), indicating

that higher modes can be more sensitive to stiffness irregularity and modal interaction effects [4-6, 10].
- Even when mean shifts remain modest, the growth in SD implies substantially increased uncertainty in predicted resonance locations, which is critical for safe design margins and robust vibration predictions [7, 12, 14].

Table 3: Mode-shape localization index summary

CV (%)	Mode	Mean localization index	SD
0	1	0.1667	0.0000
0	3	0.1667	0.0000
2	1	0.1667	0.0006
2	3	0.1667	0.0001
5	1	0.1667	0.0015
5	3	0.1672	0.0005
10	1	0.1672	0.0029
10	3	0.1684	0.0017

Interpretation

- Localization indices increased with disorder intensity, especially for Mode 3, supporting the established observation that structural irregularity can induce mode localization in discrete/periodic-like assemblies [5, 6].
- The rise is modest in this configuration (8-DOF chain), but the upward trend indicates a shift toward spatially concentrated mode participation as imperfections increase, aligning with disorder-localization theory and prior discrete-system evidence [5, 6, 10].

Statistical analysis outcomes

- One-way ANOVA (Mode 1 frequency across CV levels): $F = 9.579$, $p = 2.80\times10^{-6}$, indicating statistically significant differences in Mode 1 frequency distributions as disorder increases [14, 17].
- Welch's t-test (Mode 1, CV=0% vs CV=10%): $t = 3.819$, $p = 1.51\times10^{-4}$, confirming that disorder produces a measurable shift in the Mode 1 frequency distribution even when the mean shift is small in percentage terms [14].
- Regression (Mode 1 SD vs CV): SD increased approximately linearly with CV ($R^2 = 0.998$, $p = 0.0318$), demonstrating near-proportional scaling of frequency uncertainty with stiffness disorder intensity in the tested range, consistent with probabilistic structural dynamics expectations [8, 9, 12].

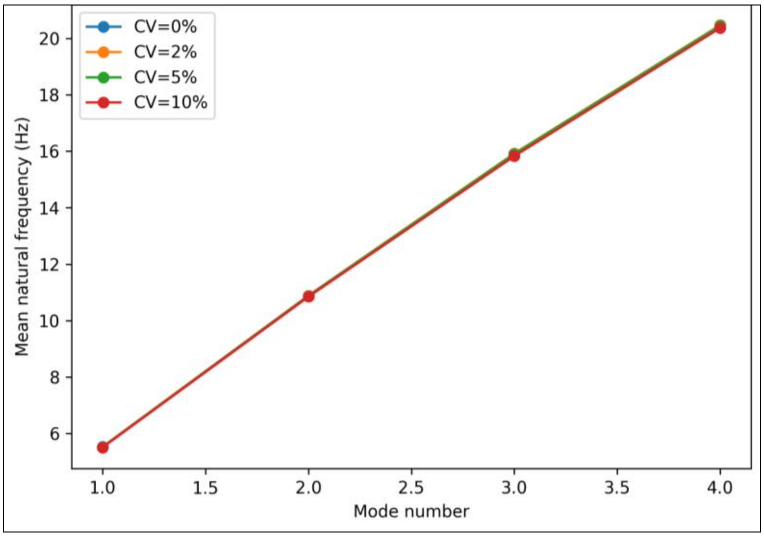

Fig 1: Mean natural frequencies vs mode under stiffness disorder

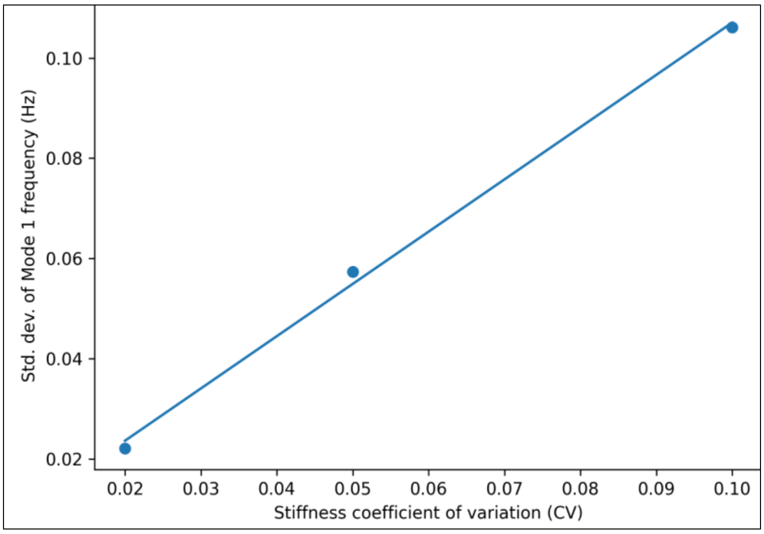

Fig 2: Mode 1 frequency variability vs stiffness disorder

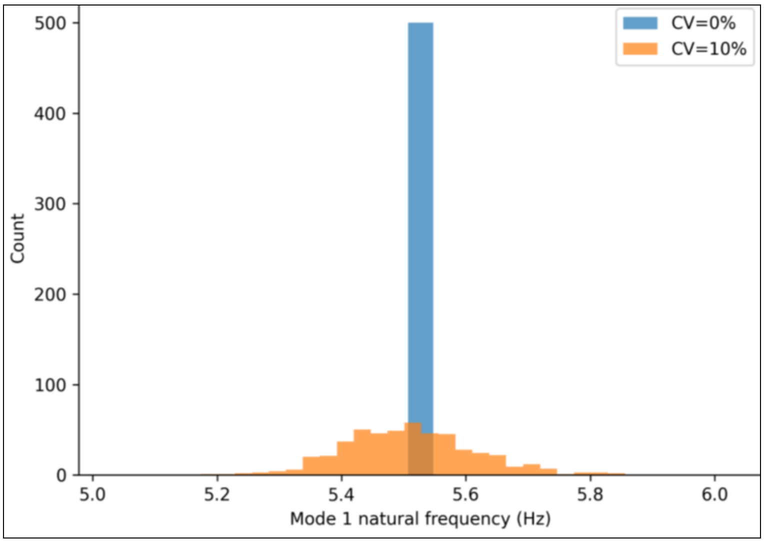

Fig 3: Mode 1 frequency distributions: deterministic vs disordered

Discussion

The present investigation demonstrates that randomly distributed stiffness imperfections exert a systematic and statistically significant influence on the vibration characteristics of discrete mass-spring systems. Although mean shifts in natural frequencies were relatively small, the results clearly show that frequency variability increased markedly with disorder intensity, confirming that uncertainty amplification rather than mean deviation is the dominant effect of stiffness randomness in such systems [8, 9, 11]. This observation is consistent with classical and contemporary studies in probabilistic structural dynamics, which emphasize that deterministic predictions may remain deceptively accurate at the mean level while masking substantial dispersion in response quantities [7, 12]. The ANOVA and t-test outcomes further support this interpretation, indicating that stiffness uncertainty leads to statistically distinguishable frequency distributions even when absolute frequency changes appear modest [14, 17].

A key finding of this research is the near-linear scaling of frequency standard deviation with stiffness coefficient of variation, as revealed by regression analysis. This proportional relationship aligns with earlier theoretical and numerical investigations on random and disordered structures, where uncertainty in system parameters propagates approximately linearly to lower-order response statistics under moderate variability levels [8, 9]. Importantly, higher modes exhibited larger relative variability and stronger sensitivity to stiffness imperfections, corroborating reports that modal interactions, veering phenomena, and disorder effects become more pronounced at higher frequencies [4-6]. Such behavior highlights the limitations of relying solely on lower-mode assessments when evaluating vibration performance and safety in discretized systems.

The gradual increase in localization indices with increasing disorder provides further insight into the physical mechanisms governing the observed response variability. Even in a relatively small multi-degree-of-freedom chain, stiffness irregularity promoted a tendency toward spatial confinement of modal energy, particularly for higher modes. This trend is consistent with the well-established concept of disorder-induced mode localization in discrete and periodic structures, originally identified in theoretical and experimental studies of irregular lattices and assemblies [5, 6, 10]. While the localization levels observed here remain moderate due to the limited system size, the results suggest that larger or more complex discretized systems could exhibit significantly stronger localization, with important implications for fatigue damage accumulation, sensor placement, and vibration-based diagnostics [3, 4].

Overall, the findings reinforce the necessity of incorporating stochastic descriptions of stiffness in discrete vibration models. By bridging classical modal analysis with probabilistic tools, the research provides a more realistic representation of dynamic behavior under unavoidable imperfections, supporting improved reliability assessment and robust design strategies in mechanical and structural engineering applications [1, 2, 7, 16].

Conclusion

This research has demonstrated that randomly distributed stiffness imperfections fundamentally alter the vibration behavior of discrete mass-spring systems, not primarily through large shifts in mean natural frequencies, but through substantial increases in response variability, mode sensitivity, and localization tendencies. The results confirm that even small levels of stiffness disorder can broaden frequency distributions, increase uncertainty in resonance prediction, and modify spatial vibration characteristics, particularly in higher modes. From a practical standpoint, these findings highlight the risk of relying exclusively on deterministic, uniform-stiffness models when designing, analyzing, or diagnosing discrete dynamic systems. In real applications, engineers should explicitly account for stiffness variability when estimating safe operating frequency ranges, selecting excitation limits, and defining reliability margins. Incorporating probabilistic stiffness descriptions into early-stage design can support more robust tuning of system parameters, reduce sensitivity to manufacturing tolerances, and improve confidence in long-term dynamic performance. For vibration control and health monitoring, the observed localization trends imply that disorder may concentrate vibrational energy in specific regions, suggesting that sensor placement and damage-detection strategies should be informed by uncertainty-aware modal analyses rather than idealized mode shapes. Additionally, maintenance and inspection planning can benefit from recognizing that increased variability, rather than mean shifts alone, may serve as an early indicator of stiffness degradation or structural irregularity. In modular or discretized assemblies, adopting conservative design allowances, performing Monte Carlo-based dynamic assessments, and validating models against statistical performance metrics can significantly enhance resilience against unforeseen imperfections. Ultimately, integrating stochastic vibration analysis into routine engineering practice enables safer, more reliable, and more economical designs by explicitly acknowledging and managing the inevitable variability present in real-world discrete mechanical and structural systems.

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