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Markov chains: Maintenance in injection molding machines

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Abstract
In businesses, it is crucial to understand the future state of assets. This research focuses on the future behavior of injection molding machines in an automotive company. The possible states of the machines are: active, decommissioned, inactive, and out of service. The objective is to present a stochastic model for predicting the future state of injection molding machines and enhance the company's decision-making regarding the maintenance policy to be applied to its machines. Prediction is achieved through a Markov chain, and forecasts of the future states of these injection molding machines were obtained. The results obtained are not promising, indicating the need for the development of new approaches and maintenance strategies to achieve the desired future state. This would enable the company to analyze its policies regarding the maintenance of injection molding machines based on its resources and customer needs.

Keywords: Markov chains, maintenance, stochastic process

Introduction

Markov chains are a stochastic method within Operations Research (OR), finding applications across various industrial domains such as marketing, sales, human resources planning, inventory management, weather forecasting, customer loyalty, among others. Therefore, this document presents some examples of their usage: finite chains in administration [1], human resources planning [2], simulation of human resources strategies [3], quality management [4], supply chain [5], and product lifecycle [6]. This method is employed for the stochastic prediction of events through the random variable of time.

On the other hand, at the beginning of the industrial era, maintenance of productive systems was conducted empirically. Then, industrial maintenance changed from a corrective approach to preventive and proactive. Markov chains have been employed in industrial maintenance for equipment repair and replacement [7], decision-making in production and maintenance [8], among other applications. Although, various techniques have been used for predictive maintenance, including vibration [9], lubricant [10], ultrasound [11], thermography analysis [12], and others.

Besides, there exist different philosophies for the restoration and preservation of productive systems, such as Total Productive Maintenance (TPM), Reliability-Centered Maintenance applied in an engine plant [13], and other asset management methodologies like Reliability Engineering, Condition-Based Maintenance Management, Execution Management of Maintenance Work, and Leadership for Reliability, with its respective techniques. If maintenance involves caring for the service provided by assets with different approaches ranging from corrective and preventive to predictive and asset management-focused [14], it is crucial to understand the future state of injection molding machines.

This study proposes to apply Markov chains to predict the future states of injection molding machines in an automotive company, utilizing the random variable of time. The goal is to optimize maintenance services by analyzing two policies about it, that confirm these machines continue to operate in an active state without failures.

Theory and Concepts

Markov chains are stochastic models used to study the evolution of certain systems or processes over repeated trials [15]. In this study, Markov chains are employed to describe the probability that a machine in an active state, functioning during a specific period, will either continue operating in the same state or transition from an active state to one with minor and major faults, or will be inactive in other period.

In this context, a stochastic process [16], is defined like a random variable $\{X_t\}$ in an indexed collection where the variable t takes values from a given set T. Frequently, T is considered as the set of non-negative integers, while X_t, represents the state of the injection molding machines and t is a period of time, that in this case, is measured in weeks. In a Markov process with n exhaustive and mutually exclusive states [17], the probabilities at a specific point in time t = 0, 1, 2, …, n are defined as in the following equation (1):

$$p_{ij} = \boldsymbol{P}\{X_T = J|X_{T-1}01\} \qquad (1)$$

Therefore, p_{ij} is the transition probability in one step, moving from state i at time t-1 to state j at time t, and is represented as a square matrix P (2).

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{21} & p_{22} & p_{23} & \cdots & p_{2n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{n1} & p_{n2} & p_{n3} & \cdots & p_{nn} \end{bmatrix} \qquad (2)$$

The matrix p defines a Markov chain and has the characteristic that its transition probabilities p_{ij}, are stationary and independent over a specified time period. To determine the absolute and n step transition probabilities, it is done using the equation number (3):

$$\boldsymbol{a_n} = \boldsymbol{a_0 P^n} \qquad (3)$$

Where α_n is the vector of the desired future state in n periods, α_0 is the initial vector of probabilities, and P^n is the n step transition matrix. From P^n, the Chapman – Kolmogorov equations [18] are established, indicating the conditional probability of transitioning from time t-1 to t in equations (4) and (5).

$$\boldsymbol{P^n} = \boldsymbol{P^{n-1}P} \qquad (4)$$

$$P^n = P^{n-m}P^m, 0 < m < n \qquad (5)$$

On the other hand, to determine the steady state probabilities of an ergodic and finite Markov chain, equations (6), (7), and (8) are used.

$$\pi_j = \lim_{n\to\infty} a_j^{(n)}, j = 0,1,2, \ldots \qquad (6)$$

Finally, it is important to know the number of expected transitions and the mean return time for the Markov chain to return to its initial state, μ_{ij}. This concept is named as the expected recurrence time to state i (9).

$$\pi = \pi P \qquad (7)$$

$$\sum_j \pi_j = 1 \qquad (8)$$

Development

The company under study specializes in manufacturing machined metal products, particularly industrial spare parts, molds, dies, and the injection of various types of plastics such as polyamides, acetals, elastomers, and others. The attention of this study is the plastic injection area. The maintenance strategy employed for injection molding machines is corrective in nature. The company don't have a preventive maintenance plan. Failures are addressed only when they occur, resulting in unstable and unreliable production, as production halts can happen at any moment.

This approach shortens the lifespan of these assets, as the current policy involves using parts from inactive injection molding machines to repair active ones and ensure production continues. The decision is made to apply Markov chains to predict the future states of injection molding machines, evaluating the current policy and comparing it with an alternative preventive approach.

Based on production and maintenance-aligned policies and corrective work orders, the possible states for injection molding machines are defined, considering their stochastic nature [19]. These states are outlined in Table 1. The desired outcome is for the assets to be in an excellent or good state, while an undesirable state would be regular or poor.

Table 1: Operational States of Injection Molding Machines

Operational State	Condition
Excellent (E)	Active without failures
Good (B)	Active with minor faults
Regular (R)	Active with major faults
Poor (M)	Inactive, not fulfilling its function

Source: Empresa Tecnología Aplicada en Maquinados S.A. de C.V

For the construction of matrix P, the initial probability vector, α_0, was created using data from the logs of the injection molding machines and maintenance work orders (both preventive and corrective) over a one-month period. The α_0 vector (10) for the four possible states of the injection molding machines is as follows:

$$a_0 = [0.1 \quad 0.4 \quad 0.3 \quad 0.2] \qquad (10)$$

Two initial policies are considered. The first one involves not repairing the inactive injection molding machines (M) and using their parts as spares for the other injection molding machines, based on historical maintenance department data and production staff experience. The transition matrix P (11) is as follows:

$$\boldsymbol{P} = \begin{array}{c} E \\ B \\ R \\ M \end{array} \begin{bmatrix} 0.5 & 0.3 & 0.2 & 0 \\ 0.2 & 0.6 & 0.15 & 0.05 \\ 0 & 0.4 & 0.4 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (11)$$

The Markov chain in graph form is shown in Figure 1, illustrating the different interactions between states according to the first policy. State B is the only one communicating with the other states, a consequence of the maintenance policy with a corrective approach to injection molding machines. To obtain the possible states, α_n, we multiply the initial probability vector by the transition matrix P^n. For example, if n = 2, the matrix operations are $\alpha_1 = \alpha_0 P^1$ y $\alpha_2 = \alpha_0 P^2$, with the following results (12).

$$a_2 = [0.143 \quad 0.353 \quad 0.1645 \quad 0.3395] \qquad (12)$$

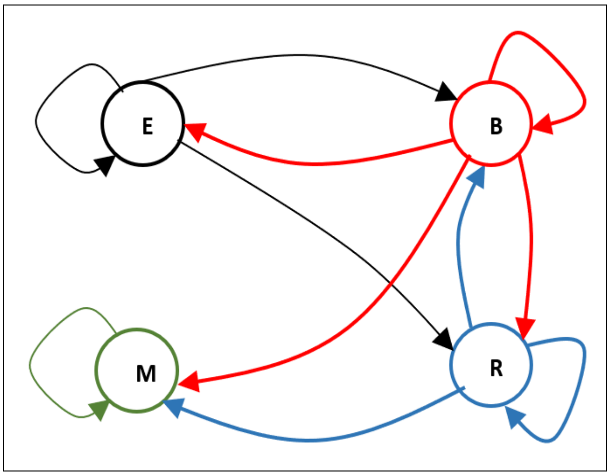

Fig 1: Graph of the first maintenance policy for injection molding machines

This indicates that after two time periods, there is a 14.3% probability that the machines will be in state E, a 35.3% probability of being in state B, a 16.45% probability of being in state R, and a 33.95% probability of being inactive.

With this stochastic method, the future state can be calculated for any period. However, to determine the long-term future state for injection molding machines, equations (13) and (14) are established to calculate the steady-state probabilities (π_i).

$$[\pi_1\ \pi_2\ \pi_3\ \pi_4]\begin{bmatrix} 0.5 & 0.3 & 0.2 & 0 \\ 0.2 & 0.6 & 0.15 & 0.05 \\ 0 & 0.4 & 0.4 & 0.2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (13)$$

$$\pi_1 = 0.5\pi_1 + 0.2\pi_2 + 0\pi_3 + 0\pi_4$$

$$\pi_2 = 0.3\pi_1 + 0.6\pi_2 + 0.4\pi_3 + 0\pi_4$$

$$\pi_3 = 0.2\pi_1 + 0.15\pi_2 + 0.4\pi_3 + 0\pi_4$$

$$\pi_4 = 0\pi_1 + 0.05\pi_2 + 0.\,2\pi_3 + 1\pi_4$$

$$\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1 \quad (14)$$

As the resulting system of equations is homogeneous and redundant, it has a unique solution. Therefore, at least one of the equations must be redundant and is eliminated [16]. So, the system of equations to solve is (15):

$$\pi_1 = 0.5\pi_1 + 0.2\pi_2 + 0\pi_3 + 0\pi_4$$

$$\pi_2 = 0.3\pi_1 + 0.6\pi_2 + 0.4\pi_3 + 0\pi_4$$

$$\pi_3 = 0.2\pi_1 + 0.15\pi_2 + 0.4\pi_3 + 0\pi_4$$

$$1 = \pi_1 + \pi_2 + \pi_3 + \pi_4 \quad (15)$$

A second policy is proposed: when an injection molding machine is inactive (M), a total maintenance should be performed to make it operate as active without failures (E). Consequently, its transition matrix P is (16):

$$\boldsymbol{P} = \begin{array}{c} E \\ B \\ R \\ M \end{array}\begin{bmatrix} 0.5 & 0.3 & 0.2 & 0 \\ 0.2 & 0.6 & 0.15 & 0.05 \\ 0 & 0.4 & 0.4 & 0.2 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (16)$$

The graph for the second strategy, transitioning from the inactive state to the active state without failures, it is shown in Figure 2. It is observed that state M communicates with state E, unlike the first policy (see Figure 1). In this policy, the company considers that when the machine is inactive, the only possible future state is total maintenance.

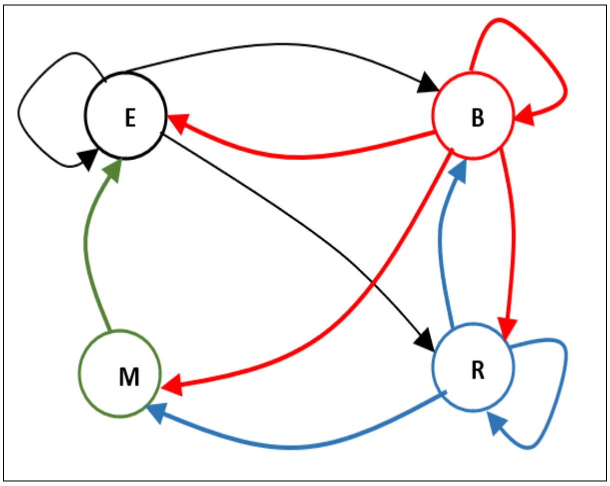

Fig 2: Graph of the second maintenance policy for injection molding machines

To determine the future states α_n with this policy, the same matrix operations are performed as in the first policy. For example, if n = 3, the matrix operations are $\alpha_1 = \alpha_0 P^1$, $\alpha_2 = \alpha_0 P^2$ y $\alpha_3 = \alpha_0 P^2$ y $\alpha_3 = \alpha_0 P^3$, with the following transition probabilities for the states (17).

$$\alpha_3 = [0.3036 \quad 0.4265 \quad 0.20835 \quad 0.06155] \quad (17)$$

Results indicate that after three periods, there is a probability that the injection molding machines will be in state E of 30.36%, in state B of 42.65%, in state R of 20.83%, and finally in state M of 6.15%. Equations for the steady-state probabilities, π_i, are also established for this policy (18) and (19).

$$[\pi_1\ \pi_2\ \pi_3\ \pi_4]\begin{bmatrix} 0.5 & 0.3 & 0.2 & 0 \\ 0.2 & 0.6 & 0.15 & 0.05 \\ 0 & 0.4 & 0.4 & 0.2 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (18)$$

$$\pi_1 = 0.5\pi_1 + 0.2\pi_2 + 0\pi_3 + 1\pi_4$$

$$\pi_2 = 0.3\pi_1 + 0.6\pi_2 + 0.4\pi_3 + 0\pi_4$$

$$\pi_3 = 0.2\pi_1 + 0.15\pi_2 + 0.4\pi_3 + 0\pi_4$$

$$\pi_4 = 0\pi_1 + 0.05\pi_2 + 0.2\pi_3 + 0\pi_4$$

$$\pi_1 + \pi_1 + \pi_1 + \pi_1 = 1 \quad (19)$$

Finally, the equations system (20) is:

$$\pi_1 = 0.5\pi_1 + 0.2\pi_2 + 0\pi_3 + 1\pi_4$$

$$\pi_2 = 0.3\pi_1 + 0.6\pi_2 + 0.4\pi_3 + 0\pi_4$$

$$\pi_3 = 0.2\pi_1 + 0.15\pi_2 + 0.4\pi_3 + 0\pi_4$$

$$1 = \pi_1 + \pi_2 + \pi_3 + \pi_4 \quad (20)$$

Results

When the system of equations for the first policy, which involves not repairing inactive injection molding machines (M), is solved with increments in n = 2, 5, 10, 15 y 20, as well as the steady-state probabilities and their mean return time, the results are shown in Table 2. It is observed that the trend of the machine active state without failures (E) is to decrease its probability, and then, it reaches zero. As a consequence, the inactive state (M) gradually increases its probability of not providing service or fulfilling its function, reaching 100% in the long term. Regarding the mean return time (μ_{ij}) for the inactive state (M) and the other states, a specific time period in weeks is not defined, as mathematically, a very large value is obtained. This indicates that with this policy, returning to state E is not possible.

Table 2: Results with the First Maintenance Policy of α_n, π_i y μ_{ij}

State	Period					π_i	μ_{ij}
	2	5	10	15	20		
Excellent	0.143	0.12635	0.08648	0.05894	0.04017	2.7E-11	∞
Good	0.353	0.27104	0.18429	0.12559	0.08559	5.8E-11	∞
Regular	0.1645	0.12528	0.08541	0.05821	0.03967	2.7E-11	∞
Poor	0.3395	0.47733	0.64382	0.75727	0.83458	1	1

The results for the second policy of performing total maintenance to function as active without failures (E) are obtained by solving the equations for future states α_n, stable states π_i, and expected recurrence time μ_{ij}, as shown in Table 3, with n = 3, 5, 10, 15 y 20. These results indicate that the probability of the excellent state tends to stabilize from period 5 onwards and in the future at 29.9. Similarly, the probability of the bad state tends to 6.3 from period 5 onwards and in the long term. On the other hand, the good state has a probability of 43.11, and the regular state has a probability of 20.73 in the future. Regarding the expected recurrence times or mean return times, μ_{ij} with the strategy of applying total maintenance to the injection machines, it takes 3.35 weeks to return to the excellent state, 2.32 periods to return to the good state, 4.82 periods to return to the regular state, and 15.87 weeks for a machine to become completely inactive or in a bad state.

Table 3: Results of α_n, π_i y μ_{ij} for injection machines with the second maintenance.

State	Period					π_i	μ_{ij}
	3	5	10	15	20		
Excellent	0.3036	0.29838	0.29851	0.29851	0.29851	0.29851	3.35
Good	0.4265	0.431	0.43118	0.43118	0.43118	0.43118	2.32
Regular	0.20835	0.20749	0.2073	0.2073	0.2073	0.2073	4.82
Poor	0.06155	0.06312	0.6302	0.06302	0.06302	0.06302	15.87

Discussion

Markov chains can be used to evaluate different maintenance strategies to ensure critical assets justify their service and function by creating various scenarios and assessing them for an efficient decision-making process. The results obtained from the analyzed policies using Markov chains indicate that the current policy employed by the company is not the most suitable. In this policy, the injection machines will not realize their function in the long term because the bad state has a 100% probability to appear. As a consequence, the probability of being in state E and the other states becomes in 0%. This mean that the decision will have to be made either to acquire new injection machines, after approximately 20 weeks, or to continue using parts from inactive machines with a 4.0% probability of being in state E and 83.46% in state M. With this, it is not possible to fulfill the maintenance function and provide service to the production systems.

The results of the second policy reveal that there is a long-term probability of 29.85% for the machines to realize their function; applying total maintenance [20] allows them to recover their service and function within a period of 3.35 weeks. There is a long-term probability of 43.12% that machines will recover to a good state in 2.32 weeks, with minor faults. There is a probability of 72.97% that the machines will realize their mission. Also exist a probability of 6.30% the state of the machines will be bad in the long term, occurring in 15.87 weeks, providing approximately 4 months to take remedial actions on the machines and recover their function.

Other factors to consider and include are the costs of preventive, corrective, and total maintenance on the injection molding machines, as this is a significant element in the analysis of different strategies for optimizing maintenance on the machines. The decision-making process will not be complete without a cost-benefit comparison. Costs are considered key success factors; therefore, maintenance policies for critical assets must be approved and supported by the company's top management.

Conclusions

The equipment in production systems is dynamic and evolving, responding to the needs of the environment. These characteristics let for the application of stochastic models to understand the possible future states in the equipment, through Markov chains. This stochastic method enables the analysis of the future state evolution of assets based on historical data and implemented policies, optimizing the decision-making process and positioning the maintenance area as a business unit.

The results obtained with the current maintenance policy of the company are neither efficient nor effective as they cause production delays, low quality levels, lack of maintenance planning, and non-compliance with promises of delivering finished products, among other factors. As indicated by the long-term results, the probability that the injection molding machines will have a future bad state is 83.46% within 20 weeks, and it will not be possible to continue using spare parts from inactive machines or repairing failures alone.

The proposed policy indicates that the desired future states of excellent and good have long-term probabilities of 29.85% and 43.12%, respectively, with a recovery time of 3.35 and 2.32 weeks each, applying a new maintenance approach compared to the first policy. Achieving the excellent state is almost impossible with the first policy. Markov chains show the probabilities of future states;

therefore, new strategies need to be implemented for the injection molding machines to perform their functions.
Finally, it should be noted that currently there are other disruptive tools of Industry 4.0 that allow real-time monitoring of machine conditions and intervention by the maintenance department to predict and prevent failures. For example, machine learning [21] for fault prediction.

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