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Bending analysis of FGM Reddy Beam Theory resting on Winkler-Pasternak elastic foundation

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Abstract
Static bending analysis of functionally graded materials FGM beam subjected to the distributed transverse load resting on elastic foundation is investigated based on Reddy beam theory RBT. Shear modulus and Young's modulus are changes in thickness direction of the beam. The governing equations of equilibrium of FGM beam are derived based on total potential energy theory. Through numerical analysis and using the Navier’s method, the maximum transverse deflection is found. Effects both spring and shear constants with power law grading index of FGM on deflection are discussed. Using numerical results from the literature, we found this paper's results to be in a good agreement. The maximum deflection increases with increase in the values of both spring and shear constants.

Keywords: Reddy Beam theory, functionally graded materials, Pasternak foundation, bending analysis

Introduction
When transverse force is applied Perpendicular to the neutral axis of structural members like beams, a bending moment is created. Transverse load, when it acts on a structure member in a perpendicular manner, is known as load acting on a structure member transversely. Depending on the volume ratio of the first phase to the second, graded materials can be obtained by layering two materials of different mechanical properties.
Rahmani *et al.* [1] derived the equations of motion. FEM use several boundary conditions to determine the maximum deflection, buckling load, and natural frequency. The numerical results demonstrate the substantial impact of the slenderness ratio. Chikh [2] analyzed the governing equations of motion by applying the theory of sinusoidal shear deformation beams and Hamilton's principle. Navier's method is utilized to calculate the maximum deflection, critical buckling load, and natural frequencies. Soltani and Asgarian [3] studied the equations of motion for a functionally graded beam. Fouad *et al.* [4] examined the bending analysis of a beam based on FGM higher order shear deformation theory supported by a Winkler elastic foundation. Navier's method is utilized to calculate the maximum deflection by numerical analytics. Yaghoobi and Torabi [5] formulated the equations of motion for Functionally Graded Material (FGM) beam. The Galerkin method is used to determine both the critical buckling load and natural frequency under various boundary conditions. Chikh [6] investigated the static bending analysis of a beam under hyperbolic shear deformation theory with a uniform distributed stress. The equilibrium governing equations are obtained by the total potential energy principle. Chen *et al.* [7] employed the Ritz approach to analyze the buckling behavior of functionally graded porous beams under various boundary conditions. Simsek [8] examined the static buckling analysis of two-dimensional functionally graded material (FGM) beams based on Timoshenko and Euler-Bernoulli beam theories under axial compressive force. The critical buckling load is determined through the application of the Ritz method under different boundary conditions. Lin *et al.* [9] studied the bending characteristics of a functionally graded beam under a uniformly distributed load on an elastic basis. Numerical methods are used to determine the maximum deflection of structures with simply supported boundary conditions. The results obtained demonstrate good agreement when compared with existing numerical data in the literature. Sayyad and Ghugal formulated the governing equations of motion for the analysis of bending, buckling, and free vibration in a functionally graded beam. Numerical findings are used to determine the buckling load, maximum deflection, and frequency by Navier's method. Avcar and Mohammed [11] examined the free vibration analysis of a functionally graded beam placed on a Winkler-Pasternak elastic basis using the Euler-Bernoulli beam theory.

The natural frequency is determined through numerical analysis under various boundary conditions. Deng *et al*. [12]. The governing equations are formulated based on Hamilton's principle for a functionally gradient beam supported by an elastic foundation. Bouazza *et al*. [13] utilized hyperbolic shear deformation theory (HYSDT) to analyze the post-buckling behavior of a thick functionally graded material (FGM) rectangular beam. The critical buckling load is determined using Navier's method for simply supported boundary conditions based on the analytical answers. Li and Batra [14] calculated the buckling load by employing the shooting method with various boundary conditions, according to FGM Timoshenko and Euler–Bernoulli beam theories. Fouda and colleagues [15] examined the equations of motion using Euler-Bernoulli beam theory. The finite element method is utilized to calculate the critical buckling load, maximum deflection, and natural frequency for a FGM Porous beam. Noori *et al*. [16] formulated the controlling equations based on FGM Timoshenko beam theory. The Complementary Functions Method is utilized to determine the maximum deflection by considering different boundary conditions. Thai and Vo [17] examined the static bending and free vibration of a functionally graded material (FGM) beam utilizing different advanced shear deformation beam theories. Determining the equations of motion through the use of Hamilton's principle. Akbaş *et al*. [18] calculated the maximum deflection and natural frequency using Navier's technique with simply supported boundary conditions. The governing equations are formulated based on Timoshenko and Euler-Bernoulli beam theories for a functionally graded material beam supported by a Winkler elastic foundation. Karamanlı formulated the governing equations for FGM beams by applying the total potential energy principle and utilizing Euler-Bernoulli, Timoshenko, and Reddy-Bickford beam theories. The FGM beam's maximum deflection is calculated using the Symmetric Smoothed Particle Hydrodynamics (SSPH) method under different boundary conditions. Vo *et al*. [20] utilized quasi-3D theory to derive the governing equations for FG sandwich beams. Finite element and Navier's methods are employed to determine the maximum deflection under various boundary conditions.

Theory and Formulations

Figure. 1 shows a functionally graded material (FGM) beam with length (L), width (b), and thickness (h). FGM beam is subjected to distributed transverse load q (x). Assumed FGM beam is made from two materials at the top of surface is metallic, Aluminum with the bottom of surface is ceramic, Alumina.

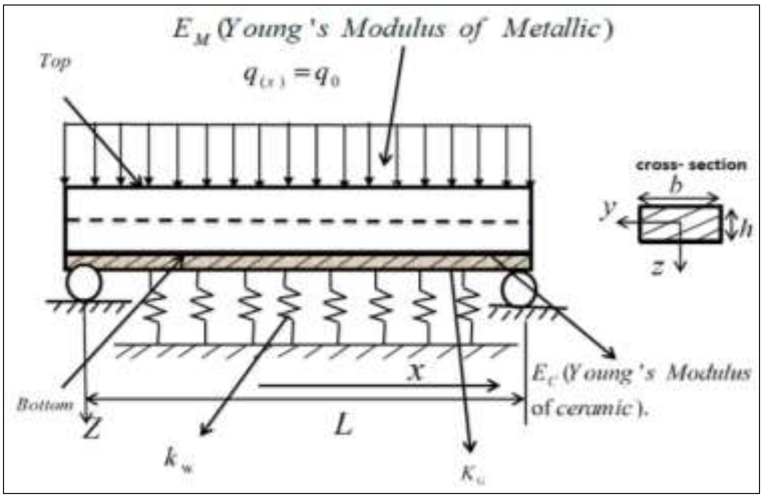

Fig 1: Functionally Graded Material (FGM) beam with length (L), width (b), and thickness (h).

Where: E_m, G_m & E_c, G_c are the material properties of the top and the bottom surfaces of the FGM beam according Eqs. (1a) & (1b) when $(z = -h/2), (E = E_m)$ and when $(z = h/2), (E = E_c)$, $(E(z)$ & $G(z))$ Variation of Young's and shear modulus of FGM.

For Reddy beam theory (RBT) the displacements can be written as:

$$U(x, y, z) = z\varphi(x) - \alpha z^3\left(\varphi(x) + \frac{\partial w(x)}{\partial x}\right) \quad (2a)$$

$$V(x, y, z) = 0 \quad (2b)$$

$$W(x, y, z) = w_0(x) \quad (2c)$$

The axial and transverse displacements of any point on the neutral axis are denoted by and, respectively. The displacement in the y direction is represented by, and signifies the transverse deflection of the beam. represents the rotation of the cross-section at any point on the neutral axis.

By using Eqs. (2a) & (2c) the axial strain and shear strain are given as:

$$\varepsilon_{xx} = \frac{\partial u(x,y,z)}{\partial x} = z\,\frac{\partial \phi}{\partial x} - \alpha z^3\left(\frac{\partial \phi}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}\right) \qquad (3a)$$

$$\gamma_{xz} = \frac{\partial u(x,y,z)}{\partial z} + \frac{\partial w(x,y,z)}{\partial x} = \phi + \frac{\partial w_0}{\partial x} - \beta z^2\left(\phi + \frac{\partial w_0}{\partial x}\right) \qquad (3b)$$

Where: $\beta = 3\alpha = 4/(h^2)$

The virtual strain energy (potential energy) of the beam can be written as:

$$U = \frac{1}{2}\int_v \sigma_{ij}\varepsilon_{ij}dV \quad , \quad \delta U = \int_v \sigma_{ij}\delta\varepsilon_{ij}dV \qquad (4)$$

The virtual strain energy (potential energy) can be written including the axial stress and shear strain of FGM Reddy beam theory as follows:

$$\delta U = \int_0^L \int_A (\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{xz}\delta\gamma_{xz})\,dA\,dx \qquad (5)$$

Where axial stress is denoted by σ, axial strain is denoted by ε, transverse shear stress is denoted by τ, shear strain is denoted by γ, the variational operator is denoted by δ, and A represents the cross-sectional area.
Substitute equations (3a) and (3c) into equation (5) and utilize equations (7a) and (7b) to calculate the ultimate strain energy (potential energy) using FGM Reddy beam theory (RBT) as follows:

$$\delta U = \int_0^L \left[-\frac{\partial}{\partial x}\left(M_x - \alpha P_x\right)\delta\phi - \alpha\frac{\partial^2}{\partial x^2}P_x\delta w_0 + \left(Q_x - \beta R_x\right)\delta\phi - \frac{\partial}{\partial x}\left(Q_x + \beta R_x\right)\delta w_0 \right] dx \qquad (6)$$

Where:

$$M_x = \int_A z\,\sigma_{xx}\,dA \;, Q_x = \int_A \sigma_{xz}\,dA \qquad (7a)$$

$$P_x = \int_A z^3\sigma_{xx}\,dA \;,\; R_x = \int_A z^2\,\sigma_{xz}dA \qquad (7b)$$

M_x is the bending moment and Q_x is the shear force force.
$P_x \,\&\, R_x$ are the higher order stress resultants.

The external work by applied forces for FGM Reddy beam theory (RBT) as follows:

$$W_{ext} = \frac{-1}{2}\int_0^L \left(F_{ext}w_0\right)dx \quad , \quad \delta W_{ext} = -\int_0^L \left(F_{ext}\delta w_0\right)dx \qquad (8)$$

Where

$$F_{ext} = F_{elastic\,foundation} + q_{(x)} \qquad (9a)$$

$$F_{ext} = -k_w w_0 + k_G\,\frac{\partial w_0}{\partial x} \qquad (9b)$$

k_w is the spring constant , k_G shear constant and $q_{(x)}$ distributed transverse load F_{ext} is the external force
The final external work for FGM Reddy beam theory (RBT) is given as:

$$\delta W_{ext} = -\int_0^L \left[\left(-k_w w_0 + k_G\,\frac{\partial^2 w_0}{\partial x^2} + q(x)\right)\delta\,w_0\right]dx \qquad (10)$$

The governing equations of FGM Reddy beam theory (RBT) using potential energy principle as follows:

$$\left(\delta U+\delta w_{ext}\right)=0 \quad (11)$$

Substituting Eqs. (6) & (10) into Eq.(11) and setting the coefficients of $\delta\varphi$ & δw_0 to zero the final equations of equilibrium of FGM Reddy beam theory (RBT) are written as follows:

$$\delta\phi: \quad -\frac{\partial}{\partial x}\left(M_x-\alpha P_x\right)+\left(Q_x-\beta R_x\right)=0 \quad (12a)$$

$$\delta w_0: \quad -\alpha\frac{\partial^2}{\partial x^2}P_x-\frac{\partial}{\partial x}\left(Q_x+\beta R_x\right)+k_w w_0-k_G\frac{\partial^2 w_0}{\partial x^2}=q(x) \quad (12b)$$

By using Hooke's law for axial and shear stresses and using Eqs. (3a) & (3b) can be written as:

$$\sigma_{xx}=E(z)\varepsilon_{xx}=E(z)\left[z\frac{\partial\phi}{\partial x}-\alpha z^3\left(\frac{\partial\phi}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}\right)\right] \quad (13a)$$

$$\sigma_{xz}=G(z)\gamma_{xz}=G(z)\left[\phi+\frac{\partial w_0}{\partial x}-\beta z^2\left(\phi+\frac{\partial w_0}{\partial x}\right)\right] \quad (13b)$$

By substituting Eqs. (13a) & (13b) into Eqs. (7a) & (7b) can be obtained as:

$$M_x=D_{xx}\frac{\partial\phi}{\partial x}-\alpha F_{xx}\left(\frac{\partial\phi}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}\right) \quad (14\text{a})$$

$$P_x=F_{xx}\frac{\partial\phi}{\partial x}-\alpha H_{xx}\left(\frac{\partial\phi}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}\right) \quad (14\text{b})$$

$$Q_x=(A_{xz}-\beta D_{xz})\left(\phi+\frac{\partial w_0}{\partial x}\right) \quad (14\text{c})$$

$$R_x=(D_{xz}-\beta F_{xz})\left(\phi+\frac{\partial w_0}{\partial x}\right) \quad (14\text{d})$$

Where:

$$\left(D_{xx},F_{xx},H_{xx}\right)=\int_A E(z)\left(z^2,z^4,z^6\right)dA \quad (15\text{a})$$

$$\left(A_{xz},D_{xz},F_{xz}\right)=\int_A G(z)\left(1,z^2,z^4\right)dA \quad (15\text{b})$$

By substituting Eqs. (14a)-(14b)-(14c)&(14d) into Eqs. (12a) & (12b) the final the governing of equations for FGM Reddy beam theory (RBT) obtained as follows:

$$-\bar{D}_{xx}\frac{\partial^2\phi}{\partial x^2}+\alpha\hat{F}_{xx}\frac{\partial^3 w_0}{\partial x^3}+\bar{A}_{xz}\left(\phi+\frac{\partial w_0}{\partial x}\right)=0 \quad (16a)$$

$$-\alpha\hat{F}_{xx}\frac{\partial^3\phi}{\partial x^3}+\alpha^2 H_{xx}\frac{\partial^4 w_0}{\partial x^4}-\bar{A}_{xz}\left(\frac{\partial\phi}{\partial x}+\frac{\partial^2 w_0}{\partial x^2}\right)+k_w w_0-k_G\frac{\partial^2 w_0}{\partial x^2}=q(x) \quad (16b)$$

Where:

$$\bar{A}_{xz} = \hat{A}_{xz} - \beta D_{xz} \quad , \bar{D}_{xx} = D_{xx} - \alpha F_{xx}$$
$$D_{xx} = D_{xx} - \alpha F_{xx} \quad , F_{xx} = F_{xx} - \alpha H_{xx}$$
$$\hat{A}_{xz} = A_{xz} - \beta D_{xz} \quad , D_{xz} = D_{xz} - \beta F_{xz}$$

Analytical solutions of bending for FGM simply supported Reddy beam theory (RBT) by using Navier-type solution method

The governing equations of the FGM Reddy beam theory (RBT) were solved for static bending using the Navier-type solution method. FGM beam under transverse load. The boundary conditions for the simply-supported Reddy beam theory (RBT) of functionally graded materials (FGM) are provided at x=0 and x=L.

$$x = 0 \Rightarrow (w = 0), (\varphi = 0), \left(\frac{\partial w}{\partial x} = 0\right), \left(\frac{\partial \varphi}{\partial x} = 0\right) \qquad (17a)$$

$$x = L \Rightarrow (w = 0), (\varphi = 0), \left(\frac{\partial w}{\partial x} = 0\right), \left(\frac{\partial \varphi}{\partial x} = 0\right) \qquad (17b)$$

The Navier-type solution method for solving the governing equations of simply-supported FGM Reddy beam theory (RBT) the variables $w_{(x)}, \varphi_{(x)}$ is defined as follows:

$$w_{(x)} = \sum_{m=1,2,3}^{\infty} W_m \sin\left(\frac{m\pi x}{L}\right) \qquad (18a)$$

$$\varphi_{(x)} = \sum_{m=1,2,3}^{\infty} \varphi_m \cos\left(\frac{m\pi x}{L}\right) \qquad (18b)$$

$$q_{(x)} = \sum_{m=1,2,3}^{\infty} q_m \sin\left(\frac{m\pi x}{L}\right) \qquad (18c)$$

Where W_m, φ_m are the unknown Fourier coefficients.

By substituting Eqs.(18a) – (18b) & (18c) into Eqs.(16a) & (16b) yields the following:

$$\left[\bar{D}_{xx}\left(\frac{m\pi}{L}\right)^2 + \bar{A}_{xz}\right]\phi_m - \left[\alpha\hat{F}_{xx}\left(\frac{m\pi}{L}\right)^3 + \bar{A}_{xz}\left(\frac{m\pi}{L}\right)\right]W_m = 0 \qquad (19a)$$

$$\left[-\alpha\hat{F}_{xx}\left(\frac{m\pi}{L}\right)^3 + \bar{A}_{xz}\left(\frac{m\pi}{L}\right)\right]\phi_m + \left[\alpha^2 H_{xx}\left(\frac{m\pi}{L}\right)^4 + \bar{A}_{xz}\left(\frac{m\pi}{L}\right)^2 + k_G\left(\frac{m\pi}{L}\right)^2 + k_w\right]W_m = q_m \qquad (19b)$$

By using Eqs. (19a) & (19b) we can find the finalized matrix configuration of FGM Reddy beam theory (RBT) as follows:

$$\begin{bmatrix} \bar{D}_{xx}\left(\frac{m\pi}{L}\right)^2 + \bar{A}_{xz} & -\alpha\hat{F}_{xx}\left(\frac{m\pi}{L}\right)^3 + \bar{A}_{xz}\left(\frac{m\pi}{L}\right) \\ -\alpha\hat{F}_{xx}\left(\frac{m\pi}{L}\right)^3 + \bar{A}_{xz}\left(\frac{m\pi}{L}\right) & \alpha^2 H_{xx}\left(\frac{m\pi}{L}\right)^4 + \bar{A}_{xz}\left(\frac{m\pi}{L}\right)^2 + k_G\left(\frac{m\pi}{L}\right)^2 + k_w \end{bmatrix} \begin{bmatrix} \phi_m \\ W_m \end{bmatrix} = \begin{bmatrix} 0 \\ q_m \end{bmatrix} \qquad (20)$$

Numerical results

The numerical results determine the static transverse deflections of the simply supported FGM utilizing Reddy beam theory (RBT) and Navier's-type solution method. The FGM beam is composed of both metallic (Aluminum) and ceramic (Alumina). The physical characteristics of the FGM beam are as follows: Length (L) = 1m, thickness (h) = 0.1m, and width (b) = 0.1m. The Poisson's ratio for metallic aluminum and ceramic alumina is comparable. The longitudinal wave number of the beam is m=1 and there is a dispersed load present.

Maximum transverse deflection of the beam is defined as:

$$\bar{w} = \frac{100 * E_m * b * h^3}{q_0 * L^4} w_0\left(\frac{L}{2}, z\right) \qquad (21)$$

Tables (1&2) shows maximum deflection w with power-law exponential n and the values of slenderness of ratio (L/h). Numerical analysis shows maximum deflection in a good agreement with both of solutions of Vo *et al.* [20] and Armağan Karamanlı [19]. It is seen from tables (1&2) that maximum deflection $\bar{w}$ increases with increasing the values of the power-law exponential (n) in FGM The flexibility of the beam increases as the power-law exponential values (n) rise, making the FGM beam closely resemble complete aluminum.

Table 1: Maximum transverse deflections of FGM Reddy beam theory (RBT) with slenderness of ratio is (L/h=5) and power-law exponential n

N	Work $\bar{w}$ RBT	Vo *et al.* [21] $\bar{w}$ RBT	Error percentage
0	3.1860	3.1397	1.4532
1	6.2977	6.1338	2.6025
2	8.0790	7.8606	2.7033
5	9.7108	9.6037	1.1029
10	10.7921	10.7578	0.3178

Table 2: Maximum transverse deflections of FGM Reddy beam theory (RBT) with slenderness of ratio (L/h=20) and the power-law exponential n

(n)	Work $\bar{w}$ RBT	Armağan Karamanlı [20] $\bar{w}$ for RBT	Error percentage
0	2.9184	2.8962	0.7607
0.5	4.1842	4.1538	0.7265
1	4.9288	4.8914	0.7588
2	5.7291	5.6799	0.8570
5	7.0390	6.9619	1.0953

Figures 2 and 3 display the impact of the shear constant KG and spring constant Kw on the maximum deflection in relation to the slenderness ratio of FGM Reddy beam theory (RBT). Figures 2 and 3 demonstrate that the maximum deflection increases with higher values of slenderness ratio and shear and spring constants, but decreases with increasing deflection.

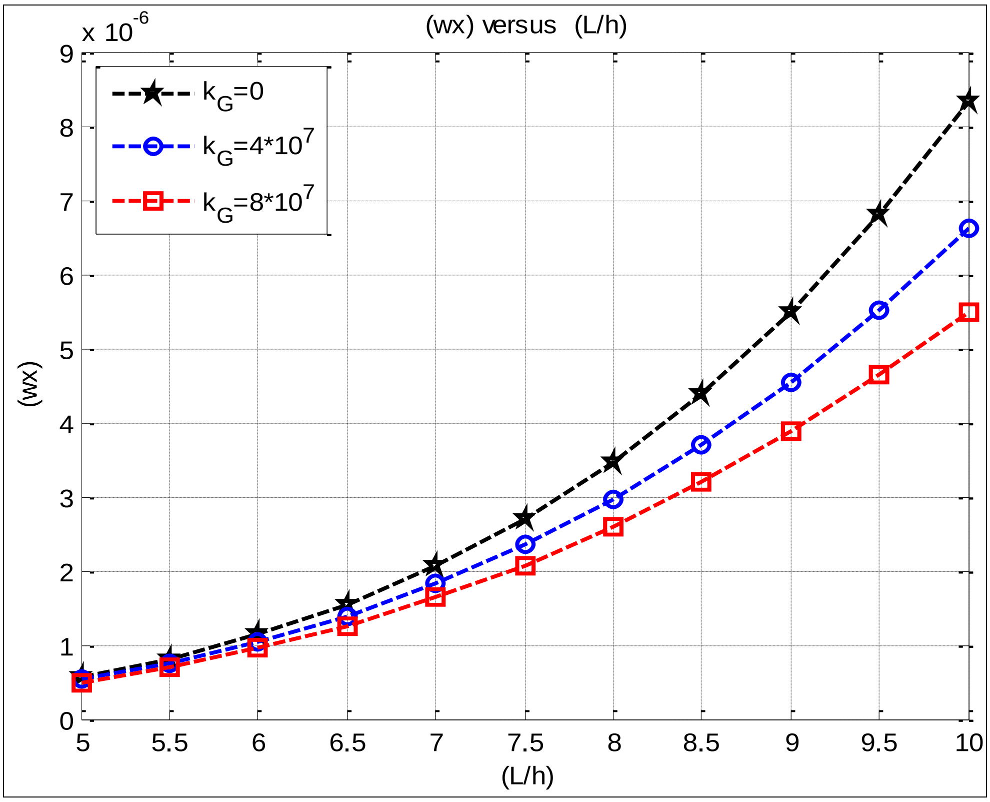

Fig 2: Present effect of the shear constant KG on the maximum deflection with slenderness ratio

Figures (4 & 5)presents the variation of the maximum deflection $\bar{w}$ with power-law index n of FGM and longitudinal wave number m. It can be seen from Figures (4 & 5) the maximum deflection increases when the slenderness ratio values increase. Figures 4 and 5 show that the greatest deflection increases as both the power-law exponential n and longitudinal wave number m grow.

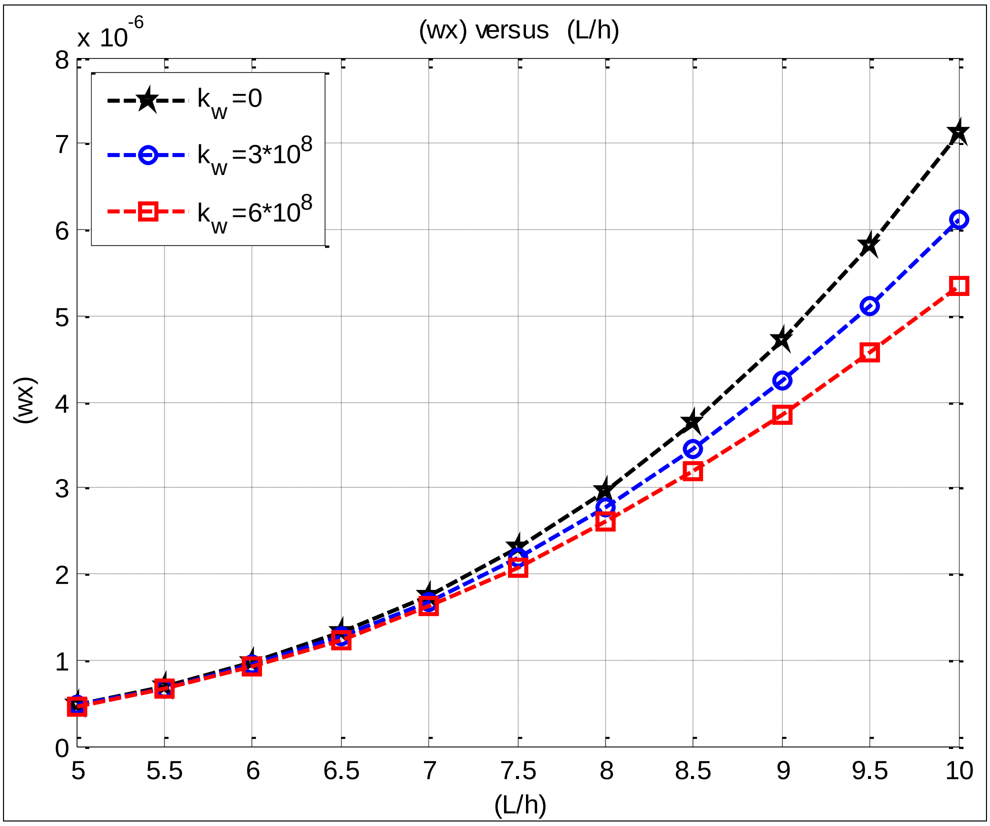

Fig 3: Present effect of the spring constant Kw on the maximum deflection with slenderness ratio

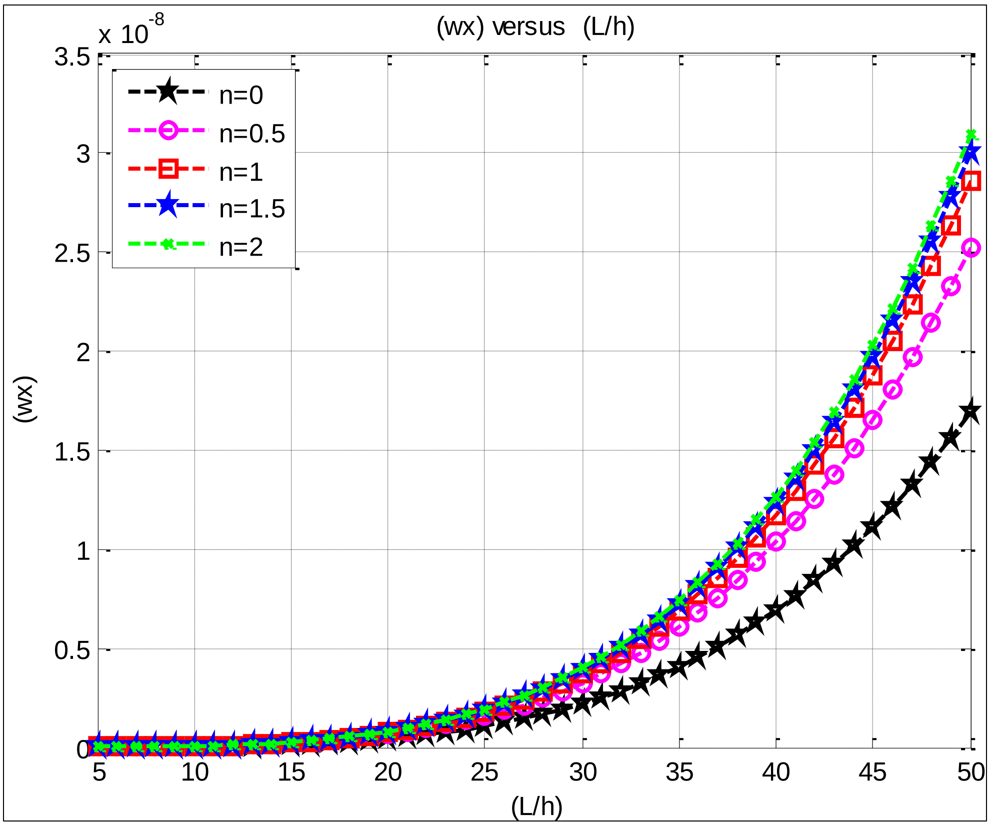

Fig 4: Effect values of the power-law of index with slenderness of ratio on the maximum deflection

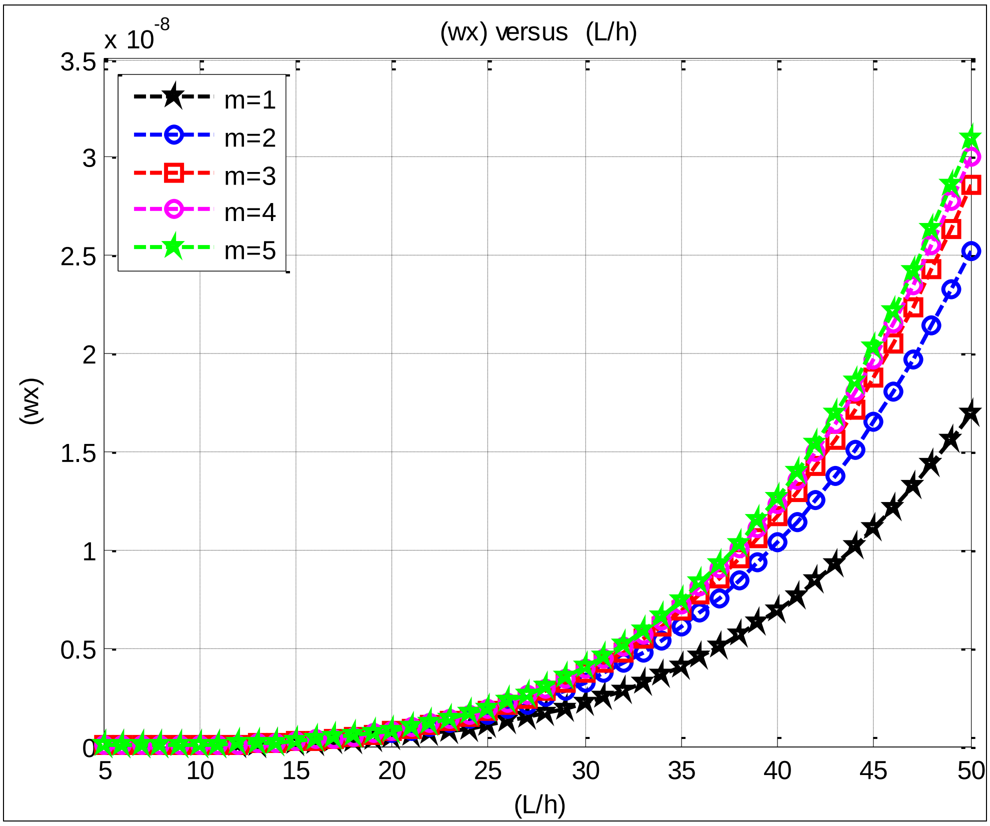

Fig 5: Effect of longitudinal wave number m with slenderness of ratio on the maximum deflection

Conclusion

Static bending analysis of functionally graded materials FGM beam subjected to the distributed transverse load resting on elastic foundation is investigated based on Reddy beam theory RBT. Shear modulus and Young's modulus are changes in thickness direction of the beam. The governing equations of equilibrium of FGM beam are derived based on total potential energy theory. Through numerical analysis and using the Navier's method, the maximum transverse deflection is found. Effects both spring and shear constants with power law grading index of FGM on deflection are discussed. Using numerical results from the literature, we found this paper's results to be in a good agreement.

1. With increase both values of the shear constant K_G and spring constant K_w the maximum deflection decreasing because with increasing in the values of spring and shear constants the beam gets stiffer. The numerical results show that the increase in values of the material power- law index n of FGM leads to the increasing the maximum deflection.
2. It can be seen from the numerical results with increasing in the values of power- law index n of FGM with longitudinal wave number m leads to increasing in the maximum deflection.
3. Numerical results shown when increase in values of slendrness ratio the maximum deflection increasing.

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