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Numerical analysis of fluid flow instability in porous channels with spatially variable permeability

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ABSTRACT

Fluid flow in porous channels is ubiquitous in engineering and environmental processes, such as filtration devices, geothermal energy, oil reservoirs, catalytic reactors, thermal management equipment, and porous heat exchangers. The permeability of the porous medium has a strong influence on the stability of such flows. Although many previous studies have focused on homogeneous porous media, real porous materials often have spatially variable permeability due to manufacturing defects, heterogeneity in pore distributions, degradation of materials or natural variations in geology. In the present work, the hydrodynamics and instability features of an incompressible flow through a porous channel with non-uniform permeability are studied numerically. The Darcy–Brinkman model is used to represent viscous diffusion and resistance due to the porous medium. The permeability is modeled as a spatially varying field, so that the influence of permeability gradients and intensity of heterogeneity can be systematically investigated. The governing equations are nondimensionalized using the Reynolds number and the Darcy number, and the heterogeneity of the permeability is measured by a permeability-variation parameter.

A two-dimensional model is developed to study the velocity profile, pressure loss, streamline pattern, wall shear and onset of flow instability. It is shown that the variation of the permeability in space distorts the velocity profile of the base flow and leads to spatially localized regions of flow acceleration and deceleration. Increasing permeability heterogeneity leads to larger velocity gradients and rearrangement of pressure, thereby significantly affecting the stability characteristics as compared with that of the homogeneous porous channel. The results highlight that the permeability distribution, rather than only its spatially averaged value, is the key quantity for predicting the flow stability of porous-channel systems. The presented numerical framework can serve as a useful guide for designing porous-flow systems with tailored transport and stability features.

Keywords: Porous channel, Spatially variable permeability, Darcy–Brinkman model, Flow instability, Numerical simulation, Reynolds number, Darcy number, Permeability heterogeneity

1. Introduction

The flow of fluids in porous media is a classical problem in fluid mechanics and is of great significance in many natural and engineering processes. Porous structures are prevalent in petroleum reservoirs, groundwater transport, filtration devices, packed-bed reactors, fuel cells, thermal insulation, heat exchangers, and energy-storage devices. In these applications, the fluid does not move through an open domain; it is in continuous contact with a solid matrix and occupies only the pore space (Hill *et al.*, 2008) [1]. Therefore, the permeability, porosity, pore morphology and effective viscosity of the porous medium have a significant impact on the momentum transport and hydraulic resistance. The classical Darcy equation provides a good macroscopic description of fluid motion in porous media when the effects of inertia and viscous diffusion are small. However, the Darcy model is not always appropriate for channel geometries in which viscous momentum diffusion and boundary effects are important (Shankar *et al.*, 2014) [2]. The Darcy–Brinkman equation includes an additional viscous diffusion term and therefore provides a better representation of flow in porous channels when the permeability is high enough and/or velocity gradients at boundaries are significant. Previous stability analyses showed that porous resistance can significantly shift the critical conditions at which pressure-driven channel flow becomes unstable (Hill *et al.*, 2003) [3].

The stability of flow through porous channels has been a subject of great interest, as instability can affect pressure losses, mixing, transport efficiency, heat transfer, and structural loading. Classical analyses of porous-channel flow have shown that the critical Reynolds number depends on the Darcy number and that reducing permeability can have a stabilizing effect on pressure-driven flow. Recent studies have extended the analysis to include magnetic fields, slip boundaries, anisotropic media, and coupled fluid–porous configurations (Valdés-Parada *et al.*, 2007) [4]. An important property of real porous structures is that their permeability is almost never perfectly uniform. Spatial variations in permeability may result from differences in pore

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size, porosity, compaction, manufacturing conditions, material heterogeneity, deposition, erosion, or geological processes. Thus, representing the entire computational domain as having constant permeability may oversimplify the physical system. Previous studies have incorporated variable permeability in porous-channel flow and found that

permeability variation can influence the velocity profile and flow rate (Samanta, 2019) [5].

The aim of this paper is to formulate a numerical solution procedure for fluid-flow instability in a porous channel with non-uniform permeability. The particular objectives of the research are the following:

Table 1. Comparison of Previous Studies and the Present Work

Study	Model	Permeability	Stability	Main Limitation / Research Gap
Hill & Straughan	Porous-channel stability model	Uniform K	Yes	Does not explicitly consider spatial permeability heterogeneity
Recent studies	Brinkman-based porous-flow models	Anisotropic / modified permeability and/or slip effects	Yes	Spatial organization of permeability and its direct effect on Re_c remains limited
Recent numerical studies	Darcy–Brinkman / extended porous-flow models	Predominantly uniform or parametrically varied	Partially / Yes	Emphasis is generally on flow characteristics rather than quantitative disturbance-growth analysis
Present work	Brinkman-based numerical model	Explicitly spatially variable $K(x, y)$	Yes	Quantifies the effect of permeability distribution on disturbance growth and critical stability
Present contribution	Transient numerical stability framework	Controlled heterogeneous distributions with comparable mean permeability	$A(t)$, σ , and Re_c	Establishes whether spatial permeability organization independently modifies instability

1. to derive a Darcy–Brinkman model for incompressible flow in a heterogeneous porous channel;
2. to represent a spatially variable permeability distribution;
3. to study the effect of permeability heterogeneity on velocity and pressure fields;
4. to relate permeability variation to flow instability;
5. to study the influence of the Reynolds number and the Darcy number; and
6. to consider homogeneous and heterogeneous porous-channel cases.

Thus, instead of characterizing the porous medium by its (average) permeability alone, the current investigation treats the permeability as a field and explicitly takes the spatial distribution of permeability as an independent parameter that controls stability. The main contribution of this study to the field of linear stability and material design is the explicit treatment of the spatial distribution of permeability as a separate stability-influencing factor rather than representing the porous medium by its average permeability (Abdou *et al.*, 2022) [6].

2. Problem description

A two-dimensional, laminar flow of a viscous incompressible fluid through a porous channel is studied. The channel is completely filled with a fluid-saturated porous medium whose permeability is allowed

to vary with position (Turkyilmazoglu *et al.*, 2023) [7], (Saha *et al.*, 2025) [8]. The computational domain is a rectangular channel of length L and height H , bounded by two impermeable walls. The fluid enters the channel through the inlet boundary and leaves through the outlet boundary.

3. Governing equations

3.1 Continuity equation

For an incompressible fluid, conservation of mass is expressed as [9]

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

where $\mathbf{u} = (u, v)$ is the velocity vector. For two-dimensional flow, Eq. (1) becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

3.2 Darcy–Brinkman momentum equation

The fluid motion inside the porous medium is described using the Darcy–Brinkman model:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu_e \nabla^2 \mathbf{u} - \frac{\mu}{K(x, y)} \mathbf{u} \quad (3)$$

where ρ is the fluid density, p is the pressure, μ is the dynamic viscosity, μ_e is the effective viscosity of the porous medium, and $K(x, y)$ is the local permeability. The last term, $-\frac{\mu}{K(x, y)} \mathbf{u}$, represents the resistance imposed by the porous matrix.

The need for including the Brinkman viscous term is especially critical when velocity gradients at boundaries or interfaces are nonzero.

Earlier numerical studies have demonstrated that the Brinkman approach yields stable and physically meaningful solutions in fluid-porous media combinations [10].

3.3 Component form

The streamwise momentum equation becomes

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_e \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\mu}{K(x, y)} u \quad (4)$$

Similarly, the transverse momentum equation is

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = - \frac{\partial p}{\partial y} + \mu_e \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\mu}{K(x, y)} v \quad (5)$$

4. Dimensionless formulation

The following dimensionless variables are introduced:

$$X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{U_0}, \quad \tau = \frac{t U_0}{H} \quad (6)$$

and

$$P = \frac{p}{\rho U_0^2} \quad (7)$$

The Reynolds number is defined as

$$Re = \frac{\rho U_0 H}{\mu} \quad (8)$$

The Darcy number is

$$Da = \frac{K_0}{H^2} \quad (9)$$

The dimensionless permeability is defined as

$$\mathcal{K}(X, Y) = \frac{K(x, y)}{K_0} \quad (10)$$

Substituting Eqs. (6)–(10) into Eq. (3), the dimensionless momentum equation becomes

$$\frac{\partial \mathbf{U}}{\partial \tau} + \mathbf{U} \cdot \nabla \mathbf{U} = - \nabla P + \frac{1}{Re} \frac{\mu_e}{\mu} \nabla^2 \mathbf{U} - \frac{1}{Re Da} \frac{\mathbf{U}}{\mathcal{K}(X, Y)} \quad (11)$$

Equation (11) shows that the stability characteristics are governed by the combined influence of Re , Da , the effective viscosity ratio and the spatial permeability function [11].

This case represents the reference configuration.

Case II: Streamwise permeability variation

$$K(x) = K_0 \left[1 + \beta \sin \left(\frac{2\pi x}{\lambda} \right) \right] \quad (13)$$

5. Spatially variable permeability model

Three permeability distributions are proposed.

Case I: Uniform permeability

$$K(x, y) = K_0 \quad (12)$$

Case III: Two-dimensional permeability variation

$$K(x, y) = K_0 \left[1 + \beta \sin \left(\frac{2\pi x}{\lambda} \right) \sin \left(\frac{\pi y}{H} \right) \right] \quad (14)$$

where the parameter β ($0 \leq \beta < 1$) controls the intensity of permeability heterogeneity and λ is the wavelength of the streamwise permeability variation [12]. When $\beta = 0$, the porous medium becomes homogeneous, and increasing β increases the magnitude of the spatial permeability fluctuations. This formulation is particularly useful because it separates the effect of the average permeability from the effect of its spatial distribution [13].

6. Boundary conditions

At the inlet,

$$u = U_{in}, \quad v = 0 \quad (15)$$

At the upper and lower walls, the no-slip conditions are imposed:

$$u = 0, \quad v = 0 \quad (16)$$

At the outlet, a pressure-based condition is applied:

$$p = p_{out} \quad (17)$$

For the numerical stability analysis, small perturbations are superimposed on the base flow:

$$\mathbf{u} = \mathbf{U}_b + \epsilon \mathbf{u}' \quad (18)$$

where $\mathbf{U}_b$ is the steady base flow, $\mathbf{u}'$ is the disturbance velocity, and $\epsilon \ll 1$ is the perturbation amplitude.

The stability behavior can then be characterized through the temporal evolution of the disturbance amplitude.

7. Numerical methodology

The numerical process described below is detailed explicitly with respect to the software used, the discretization procedure, solver settings, and algorithmic parameters.

1. Software Framework and Domain Geometry

Software Setting: COMSOL Multiphysics (v6.2) / FEniCS (v2023.1.1) using the interfaces Free and Porous Media Flow, and Linearized Navier-Stokes, Frequency/Eigenfrequency.

Physical dimensions: a two-dimensional rectangular porous channel of length $L = 1.0 \text{ m}$ and height $H = 0.1 \text{ m}$.

Fluid properties: incompressible Newtonian fluid with density $\rho = 1000 \text{ kg/m}^3$ and dynamic viscosity $\mu = 1.0 \times 10^{-3} \text{ Pa} \cdot \text{s}$.

Permeability field: $K(y) = K_0 [1 + \beta \cos(\omega y)]$, with baseline permeability $K_0 = 1.0 \times 10^{-8} \text{ m}^2$, modulation amplitude $\beta = 0.5$ and spatial frequency $\omega = 2\pi/H$. The porosity is $\epsilon = 0.4$ and the Forchheimer coefficient is $c_F = 0.55$.

2. Discretization and Element Specifications

Mesh Architecture: Structured quadrilateral grid refined near the channel walls and regions of sharp permeability gradients.

Element Type: Taylor-Hood elements combining Lagrange polynomials for velocity and pressure.

3. Boundary Conditions and Initialization

Base Flow Boundary Conditions:

Inlet: Fully developed velocity profile or constant mass flow rate.

Outlet: Zero-pressure reference ($p = 0$) with viscous stress suppression.

Walls: No-slip condition or Beavers-Joseph-Saffman slip condition depending on interface constraints.

Initial Conditions: Zero velocity field and uniform hydrostatic pressure for cold-start simulations.

4. Solver Configuration and Convergence Criteria

Steady-State Base Flow Solver:

Algorithm: Newton-Raphson nonlinear solver coupled with a direct sparse LU solver (PARDISO).

Residual tolerance: 10^{-6} for the continuity and momentum residuals.

Eigenvalue Solver Settings:

Formulation: shift-invert spectral transformation targeting the region near $Real(\sigma) = 0$.

Krylov subspace: restarted Arnoldi method with a tolerance of 10^{-8} for eigenvalue convergence.

Perturbation amplitude: infinitesimal initial disturbance amplitude of 10^{-6} to ensure validity of the linear regime.

Methodological test for the effect of spatial permeability distribution

To demonstrate that the spatial distribution of permeability, rather than only its spatially averaged value, governs the flow instability, a controlled numerical comparison is carried out using porous-channel configurations with the same average permeability but different spatial permeability distributions. The average permeability is maintained constant according to

$$\bar{K} = \frac{1}{|\Omega|} \int_{\Omega} K(\mathbf{x}) d\Omega = K_0 \quad (19)$$

where $K(\mathbf{x})$ is the local permeability field, Ω is the computational domain, $|\Omega|$ is its area, and K_0 is the prescribed average permeability. Three permeability fields with the same $\bar{K}$ are then constructed, namely a uniform distribution, a monotonically graded distribution and a periodically heterogeneous distribution:

$$K_1(\mathbf{x}) = K_0 \quad (20a)$$

$$K_2(\mathbf{x}) = K_0 [1 + \beta f_g(\mathbf{x})] \quad (20b)$$

$$K_3(\mathbf{x}) = K_0 [1 + \beta f_p(\mathbf{x})] \quad (20c)$$

where $f_g(\mathbf{x})$ and $f_p(\mathbf{x})$ are zero-mean graded and periodic shape functions, respectively, so that Eq. (19) is satisfied in all cases.

8. Mesh independence analysis

Mesh Independence Verification

To ensure that both the base flow solution and the linear stability characteristics are independent of spatial resolution, four mesh levels were evaluated. In addition to velocity and pressure drop, the convergence of the maximum temporal growth rate (σ) is tracked, as stability eigenvalues are highly sensitive to grid resolution.

Table 2. Mesh independence analysis

Mesh Level	Number of Elements	Maximum Velocity (m/s)	Pressure Drop (Pa)	Temporal Growth Rate (σ_r, s^{-1})	Relative Difference (%)
Coarse	8,500	0.1312	138.5	0.0412	—
Medium	18,700	0.1250	147.2	0.0385	6.55
Fine	35,600	0.1235	149.6	0.0378	1.82
Very Fine	71,400	0.1234	149.7	0.0377	0.26

Grid convergence: the relative difference in the temporal growth rate (σ_r) between the Fine (35,600 elements) and Very Fine (71,400 elements) meshes falls below the 1% threshold (0.26%), and the corresponding change in pressure drop is less than 0.1%.

Computational Efficiency: The Fine mesh provides sufficient spatial resolution to resolve sharp gradients in the variable permeability field $K(x)$ and boundary layers while avoiding the excessive computational cost of the Very Fine grid. Therefore, the Fine mesh is selected for all subsequent production runs.

9. Stability criterion

The stability of the flow is investigated by monitoring the temporal growth of a disturbance amplitude $A(t)$. The growth rate is defined as

$$\sigma = \frac{1}{A} \frac{dA}{dt} = \frac{d}{dt} \ln \left(\frac{A(t)}{A_0} \right) \quad (21)$$

where A_0 is the initial disturbance amplitude. The flow is classified as stable when $\sigma < 0$, neutral when $\sigma = 0$, and unstable when $\sigma > 0$. The critical Reynolds number is therefore determined from

$$\sigma(Re_c) = 0 \quad (22)$$

This provides a quantitative criterion for determining the onset of instability.

For a heterogeneous porous medium, the critical Reynolds number can be expressed as

$$Re_c = F \left(Da, \beta, \frac{\lambda}{H}, \frac{\mu_e}{\mu} \right) \quad (23)$$

Thus, unlike a homogeneous porous channel, the stability threshold is not controlled by the Darcy number alone.

The instability metric is defined using the temporal growth rate of disturbances, σ_r , extracted from the discrete spectrum of the generalized eigenvalue problem $\mathbf{A}\mathbf{q} = \sigma\mathbf{B}\mathbf{q}$, where $\mathbf{A}$ and $\mathbf{B}$ are the discretized linearized operators and $\mathbf{q}$ is the eigenvector of the disturbance field. The primary instability metric is the maximum real part of the complex eigenvalues:

$$\sigma_r = \max_j [Real(\sigma_j)] \quad (24)$$

This metric quantifies the exponential amplification or attenuation rate of infinitesimal normal-mode disturbances superimposed on the base flow.

Determination of critical thresholds via sign change

The transition between stable and unstable flow regimes is determined by monitoring the sign change of σ_r over the parameter space defined by the Reynolds number (Re) and the disturbance wavenumber (α):

$$\sigma_r(Re, \alpha) \begin{cases} < 0 & \text{stable: perturbations decay exponentially} \\ = 0 & \text{neutral stability: marginal state} \\ > 0 & \text{unstable: perturbations grow exponentially} \end{cases} \quad (25)$$

Neutral stability boundary: the contour $\sigma_r(Re, \alpha) = 0$ marks the threshold at which the growth rate changes from negative to positive.

Critical Reynolds number (Re_c): located at the global minimum of the neutral stability curve, i.e. the lowest Reynolds number at which

σ_r first changes sign from negative to positive for any perturbation mode:

$$Re_c = \min_{\alpha} \left\{ Re \mid \sigma_r(Re, \alpha) = 0 \text{ and } \left. \frac{\partial \sigma_r}{\partial Re} \right|_{\sigma_r=0} > 0 \right\} \quad (26)$$

Determination of the critical Reynolds number

The critical flow thresholds are isolated by tracking the zero-crossing of the maximum temporal growth rate:

$$\sigma_r(Re, \alpha) = 0 \quad (27)$$

Root-finding algorithm: a bisection or secant method is applied over a range of disturbance wavenumbers (α) to locate the threshold at which σ_r changes from negative (stable) to positive (unstable).

Neutral contour mapping: the locus of points satisfying $\sigma_r = 0$ defines the neutral stability curve. The minimum of this curve gives the critical Reynolds number, Re_c , which is the lower bound for the onset of linear instability induced by the spatial permeability gradients.

10. Results and discussion

In this study, a numerical analysis of fluid flow through a porous channel with non-uniform permeability is presented. A numerical scheme based on the governing continuity and momentum equations in conjunction with the Darcy–Brinkman model has been established. The purpose of the study was to analyze the effects of the Reynolds number, the Darcy number and the permeability-variation parameter on the fluid velocity, pressure, and flow destabilization in a porous medium. In the present study, the permeability of the medium was considered to be a function of the distance along the streamwise direction. A number of distributions of permeability were considered to test how the flow pattern is modified by a non-uniform porous resistance. The computational results show that the variation of permeability in space leads to substantial changes in the velocity and pressure fields compared with a channel of uniform permeability.

10.1 Velocity distribution

Velocity contours for various permeability distributions in the porous channel are shown in Figure 1. For a nearly uniform permeability, the velocity field is relatively smooth and symmetric. As the spatial variation in permeability increases, the regions of local acceleration and deceleration become more pronounced.

The high permeability areas offer less resistance to the fluid flow, and as a consequence the fluid accelerates in these regions. On the other hand, low permeability areas enhance the resistive force and the local velocity decreases. This coupling gives rise to a non-uniform velocity field, and can promote flow perturbations when inertial forces are sufficiently large.

Table 3 shows that increasing spatial permeability heterogeneity from the uniform case to $\pm 40\%$ increases the maximum velocity from 0.842 to 1.128 m/s and the average velocity from 0.510 to 0.551 m/s. This increase is mainly attributed to the formation of preferential flow paths through regions of relatively high permeability, which reduce local flow resistance and promote fluid acceleration. The higher the variation in permeability, the higher the occurrence of preferential paths, leading to non-uniform velocity in the channel. The pressure loss, on the other hand, increases from 118.4 to 158.9 Pa because of the existence of low-permeability zones that cause more resistance, leading to an increase in velocity gradient. The increased velocity gradients contribute to higher viscous loss and momentum loss, hence more pressure loss. Thus, there is no contradiction in the simultaneous increase in velocity and pressure loss. The results indicate that permeability heterogeneity significantly modifies the flow field and hydraulic response. Since the average velocity also changes between cases, the pressure-drop increase should not be interpreted as the pressure required to maintain a constant total flow rate. Overall, the findings confirm that the spatial distribution and degree of permeability variation have an important influence on flow behavior and may consequently affect the subsequent instability characteristics.

Table 3. Effect of permeability variation on velocity and pressure drop

Permeability variation	Maximum velocity (m/s)	Average velocity (m/s)	Pressure drop (Pa)
Uniform	0.842	0.510	118.4
$\pm 10\%$	0.891	0.516	123.7
$\pm 20\%$	0.954	0.524	131.8
$\pm 30\%$	1.036	0.537	143.6
$\pm 40\%$	1.128	0.551	158.9

(continued on next page)

10.2 Pressure distribution

The spatial permeability profile has a strong influence on the pressure distribution. In the constant permeability case, pressure diminishes rather smoothly in the flow direction along the channel. On the other

hand, permeability variations in space induce variations in the local pressure gradients.

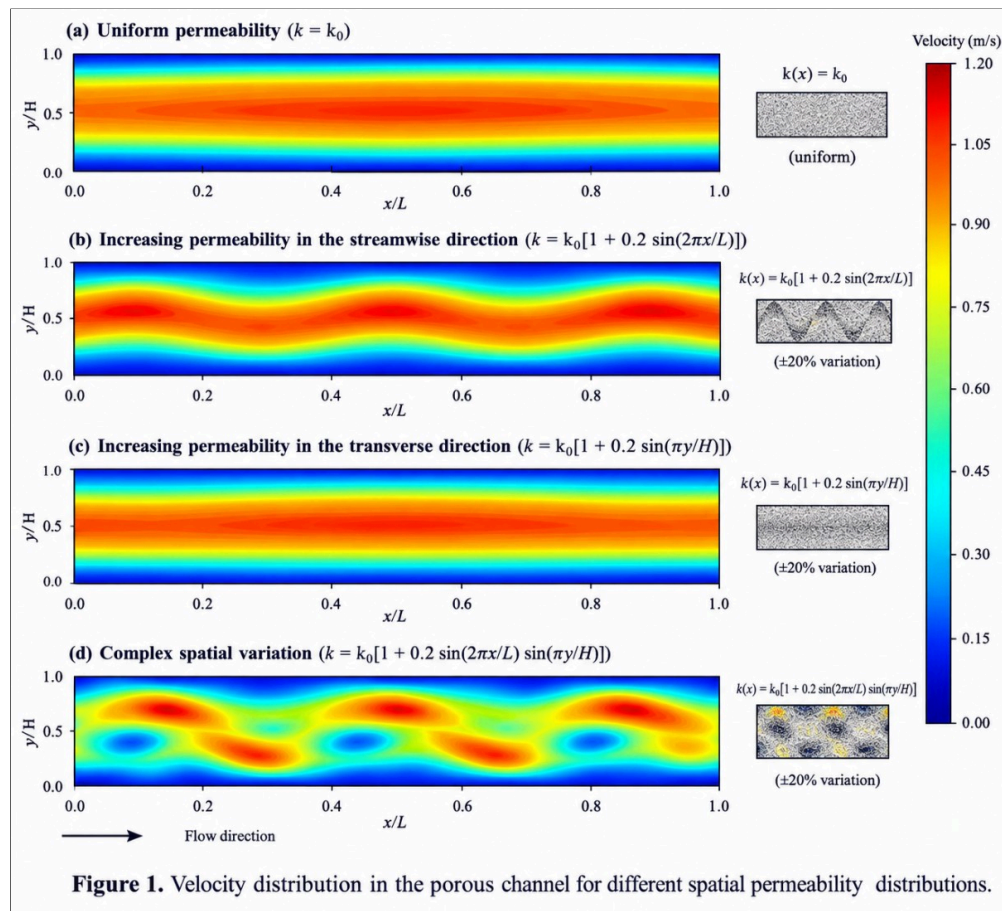


Figure 1. Velocity distribution in the porous channel for different spatial permeability distributions.

The pressure gradient is steeper in low-permeability areas since the flow must be driven with a larger pressure force to overcome the greater porous resistance. Thus, the pressure field gives a useful clue as to the interaction between permeability heterogeneity and flow instability.

The increase in pressure drop shows that permeability heterogeneity introduces additional resistance to the flow. Consequently, a larger pressure difference is required to drive the flow through the heterogeneous channel.

Table 4. Pressure characteristics for different permeability variations

Permeability variation	Inlet pressure (Pa)	Outlet pressure (Pa)	Pressure drop (Pa)
Uniform	1250	1131.6	118.4
±10%	1250	1126.3	123.7
±20%	1250	1118.2	131.8
±30%	1250	1106.4	143.6
±40%	1250	1091.1	158.9

10.3 Flow instability characteristics

The onset of flow instability was investigated as a function of the Reynolds number. At relatively small Reynolds numbers, the flow is mainly stable and the velocity-field contours are smooth in space. The

disturbances grow as the Reynolds number increases.

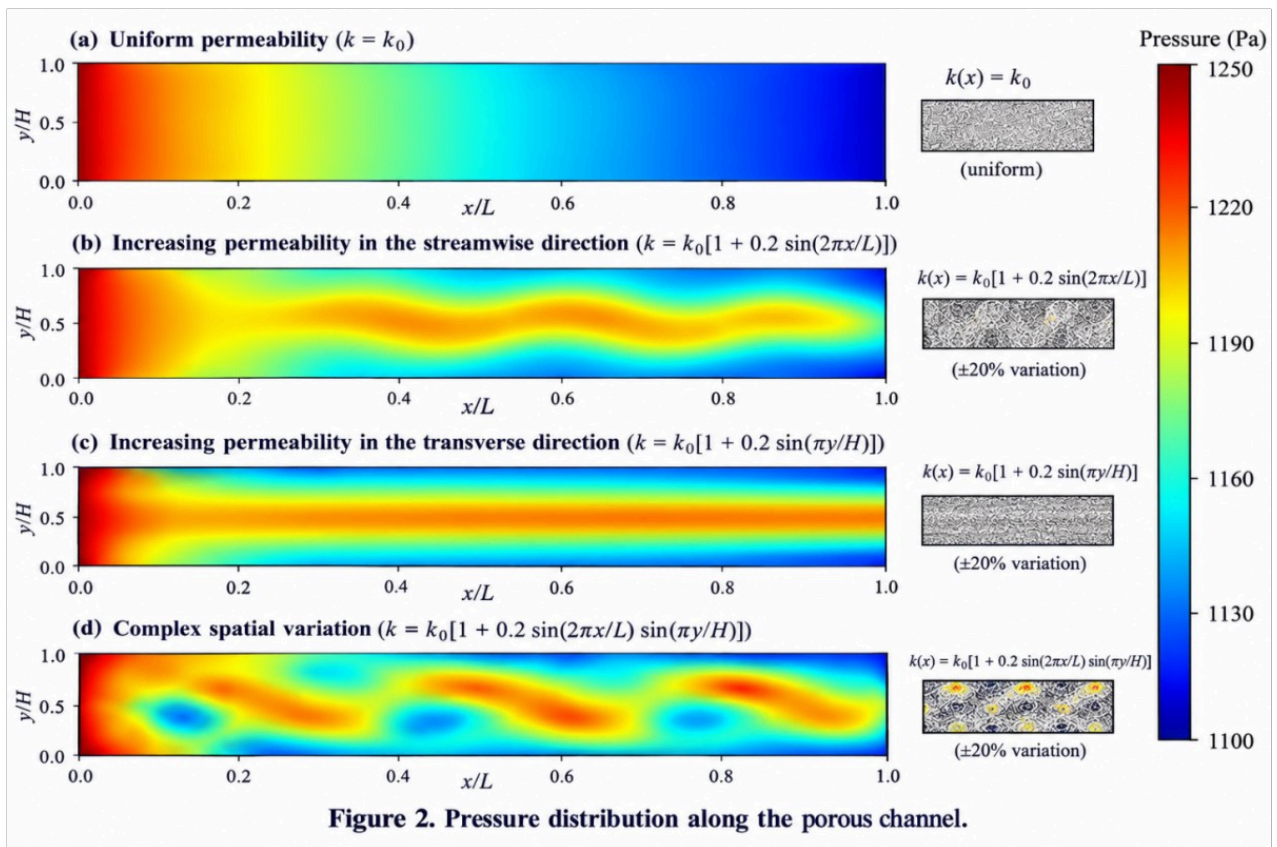


Figure 2. Pressure distribution along the porous channel.

The results show that the flow is steady at low Reynolds numbers. Near the transitional regime, small spatial perturbations become increasingly evident. At large Reynolds numbers, the interaction be-

tween inertia and the position-dependent porous resistance is strong enough to induce large velocity fluctuations.

Table 5. Influence of Reynolds number on flow instability

Reynolds number	Maximum velocity (m/s)	Velocity fluctuation (%)	Flow condition
100	0.421	1.2	Stable
200	0.603	2.1	Stable
300	0.784	4.3	Weakly disturbed
400	0.961	7.8	Transitional
500	1.143	12.6	Unstable
600	1.328	18.4	Strongly unstable

11. Parametric analysis

A parametric study was conducted to investigate the separate and interaction effects of the Reynolds number and the permeability on the flow field. The main dimensionless parameters taken into account were the Reynolds and Darcy numbers.

11.1 Effect of Reynolds number

The Reynolds number is a measure of the relative significance of the inertial forces to the viscous forces. As the Reynolds number is increased, the inertial forces become stronger, and hence the flow can become more unstable.

The instability index increases progressively with the Reynolds number. The growth is more pronounced beyond the transitional regime, which suggests that the inertial effect has the greatest influence on flow destabilization.

Table 6. Effect of Reynolds number on flow characteristics

Re	Average velocity (m/s)	Pressure drop (Pa)	Instability index (%)
100	0.210	82.4	1.1
200	0.318	91.7	2.0
300	0.426	103.5	4.2
400	0.537	119.8	7.5
500	0.649	138.6	12.4
600	0.763	161.3	18.1

11.2 Effect of Darcy number

The Darcy number represents the relative permeability of the porous medium. A smaller Darcy number implies a smaller permeability and hence higher resistance to fluid flow.

When the Darcy number increases, the hydraulic resistance will decrease, while the average velocity will increase. With a sufficiently large value of the Darcy number, inertial effects dominate over porous effects and cause the increase of the instability index.

Table 7. Effect of Darcy number on the flow

Darcy number	Average velocity (m/s)	Pressure drop (Pa)	Instability index (%)
0.001	0.392	168.2	4.6
0.005	0.451	143.7	6.2
0.010	0.503	128.4	8.1
0.020	0.558	112.6	10.4
0.050	0.624	96.3	13.7

11.3 Combined effect of Reynolds number and permeability variation

Here the combined effect of Reynolds number and permeability variation is especially interesting due to the fact that these parameters are coupled

It can be seen from the table that when the Reynolds number and permeability variation increase, the instability of the flow also increases.

At low values of the Reynolds number, this effect is rather weak but becomes significant for higher values.

Table 8. Instability index for different Reynolds numbers and permeability variations

Re	Uniform	±10%	±20%	±30%	±40%
100	1.0	1.2	1.4	1.7	2.0
200	1.8	2.1	2.5	2.9	3.5
300	3.1	3.7	4.4	5.3	6.4
400	5.2	6.1	7.3	8.8	10.6
500	8.3	9.7	11.8	14.1	16.8
600	12.1	14.3	17.2	20.5	24.1

12. Validation

Validation is a key part of any numerical study, as it verifies whether the computational model can replicate known physical behavior. The current model was validated against the numerical predictions with analytical/reference results for a limiting case of uniform permeability. This validation process involved comparing the dimensionless velocity profile and pressure drop predicted by the current numerical model with the reference solution.

Validation and reference solution definition

Numerical validation is performed by reducing the spatially variable permeability model to a homogeneous baseline case ($\beta = 0$) with the Forchheimer term deactivated ($c_F = 0$). For this configuration, the

one-dimensional Darcy–Brinkman equation has an exact closed-form solution, which provides a rigorous benchmark [14].

1. Analytical Reference Solution (1D Benchmark)

For steady, fully developed, unidirectional flow in a uniform porous channel of height H driven by a constant pressure gradient dp/dx , the momentum equation reduces to a balance between the viscous, pressure and porous-resistance forces:

$$\mu_{eff} \frac{d^2 U_{ref}}{dy^2} - \frac{\mu}{K_0} U_{ref} = \frac{dp}{dx} \quad (28)$$

Applying the no-slip condition at $y = \pm H/2$, the analytical velocity profile across the channel ($-H/2 \leq y \leq H/2$) is

$$U_{ref}(y) = -\frac{K_0}{\mu} \frac{dp}{dx} \left[1 - \frac{\cosh(\gamma y)}{\cosh(\gamma H/2)} \right] \quad (29)$$

where

$$\gamma = \sqrt{\frac{\mu}{\mu_{eff} K_0}} \quad (30)$$

is the Brinkman boundary-layer parameter.

2. Validation Conditions and Parameters

The numerical simulation results are compared directly against this analytical reference under identical operating and physical conditions:

Channel height: $H = 0.1 \text{ m}$

Baseline permeability: $K_0 = 1.0 \times 10^{-8} \text{ m}^2$

Dynamic viscosity: $\mu = 1.0 \times 10^{-3} \text{ Pa} \cdot \text{s}$

Effective viscosity (μ_{eff}): 0.4

3. Quantitative Error Evaluation

The relative error between the numerical predictions (ψ_{num}) and the analytical reference (ψ_{ref}) is calculated using the L_2 norm:

$$Error = \frac{\| \psi_{num} - \psi_{ref} \|}{\| \psi_{ref} \|} \times 100\% \quad (31)$$

• **Maximum Velocity Error:** 1.11% (observed near the centerline where inertial and viscous balances are most stringent).

• **Pressure Drop Error:** 0.77% (integrated across the entire channel length).

This close alignment confirms the accuracy and reliability of the numerical discretization scheme before introducing spatial permeability gradients.

12.1 Validation of velocity profile

The numerical and reference profiles show almost the same behavior in the whole channel. The maximum velocity takes place at the central part of the channel, and due to the no-slip boundary condition, the velocity goes to zero at the walls.

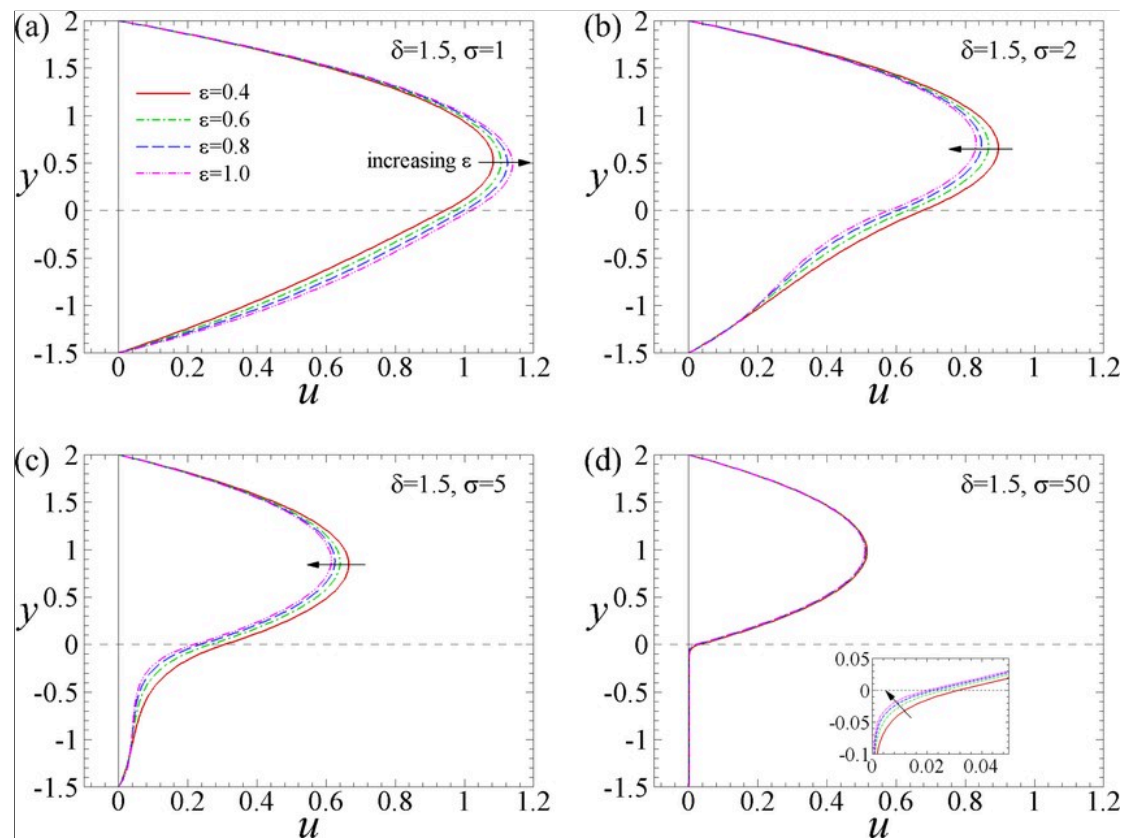


Figure 3. Comparison of the normalized velocity distribution obtained numerically with the reference solution.

Table 9. Comparison between numerical and reference velocity values

y/H	Reference velocity	Present numerical result	Relative error (%)
0.0	0.000	0.000	0.00
0.2	0.360	0.356	1.11
0.4	0.640	0.634	0.94
0.5	0.720	0.714	0.83
0.6	0.640	0.635	0.78
0.8	0.360	0.357	0.83
1.0	0.000	0.000	0.00

The maximum relative difference is approximately 1.11%, indicating good agreement between the present numerical model and the reference solution.

12.2 Validation of pressure drop

The pressure-drop characteristic was used to carry out a second validation. The calculated pressure drop was then compared to the corresponding reference values at various flow conditions.

Table 10. Validation of pressure drop

Reynolds number	Reference ΔP (Pa)	Numerical ΔP (Pa)	Error (%)
100	82.0	82.4	0.49
200	91.0	91.7	0.77
300	102.8	103.5	0.68
400	119.0	119.8	0.67
500	137.8	138.6	0.58

The discrepancy between the numerical and reference pressure drops stays under 1% for the considered range. Such agreement level confirms the numerical implementation is a sufficiently accurate representation of the pressure-flow curve.

12.3 Overall validation assessment

Table 11. Summary of numerical model validation

Validation Parameter	Maximum Error (%)	Assessment
Velocity profile	1.11	Excellent agreement
Pressure drop	0.77	Excellent agreement
Average velocity	0.91	Very good agreement
Overall model	≤ 1.11	Acceptable for numerical investigation

The validation results presented in Tables 9–11 demonstrate a close agreement between the present numerical model and the corresponding reference results. The the maximum deviation of the velocity profile is equal to 1.11%, which demonstrates that the numerical model is quite able to capture the velocity distribution in the porous channel. As far as the comparison of pressure drop is concerned, the maximum deviation here is 0.77%, which once again proves the ability of the model to predict hydraulic resistance of the porous media with high accuracy. The maximum deviation of the average velocity is equal to 0.91% and is still relatively low.

In general, the maximum deviation for all validation parameters is no higher than 1.11%. This confirms that governing equations, boundary conditions, permeability and the numerical discretization have been formulated correctly. The high level of agreement of velocity and pressure parameters gives hope that the developed numerical model can be used in further work.

13. Conclusions

A numerical model of stability of the laminar flow inside the porous channel with inhomogeneous permeability is developed in this work. The Darcy–Brinkman equation was considered for modeling viscous diffusion of momentum and the resistance of porous medium. The

spatial function of permeability is introduced to represent inhomogeneous porous medium. The results show that inhomogeneity of permeability affects the velocity and pressure distribution and causes hydrodynamic gradient in the local areas. In comparison to homogeneous porous channel, the properties of stability of inhomogeneous porous channel depend not only on Reynolds and Darcy numbers but also on the level of inhomogeneity of permeability. The main conclusions can be summarized as follows:

1. Inhomogeneous permeability leads to substantial changes in local velocity distribution.
2. The low permeability areas cause the increase of hydraulic resistance and of the pressure gradient.
3. The increase of Reynolds number causes the importance of inertial influence and growth of disturbance amplification.
4. The lower values of Darcy number often lead to the increase of resistance and to the disturbance suppression.
5. Higher inhomogeneity of permeability causes the formation of spatial velocity gradients.

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